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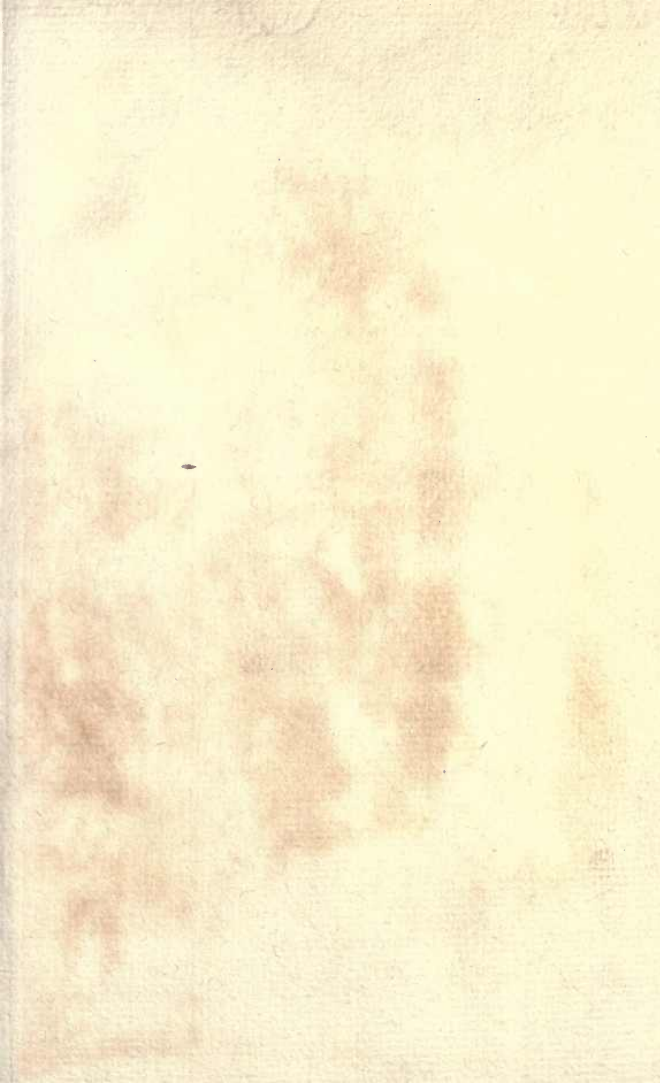
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C. K. OGDEN



OF THE
LAWS of CHANCE,
OR, A
METHOD

Of Calculation of the
Hazards of GAME,
Plainly demonstrated,
And applied to GAMES at present
most in Use ;

Which may be easily extended to the most
intricate Cases of CHANCE imaginable.

The FOURTH EDITION,
Revis'd by JOHN HAM.

By whom is added,
A Demonstration of the Gain of the Banker in any
Circumstance of the Game call'd PHARAON ;
And how to determine the Odds at the
ACE of HEARTS or FAIR CHANCE ;
With the Arithmetical Solution of some
Questions relating to LOTTERIES ;
And a few Remarks upon HAZARD and
BACKGAMMON.

L O N D O N :

Printed for B. MOTTE and C. BATHURST, at the
Middle-Temple Gate in Fleet-street. M.DCC.XXXVIII.

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With the Mathematical Solution of many
Questions relating to LOTTERIES;
And a new Method of Hazard, and
Backgammon.

LONDON:

Printed by J. Bland, and C. B. Bland, at the
Printers, in Pall-mall, near the Theatre.



P R E F A C E.

I*T is thought as necessary to write a Preface before a Book, as it is judg'd civil, when you invite a Friend to Dinner, to proffer him a Glass of Hock beforehand for a Whet: And this being maim'd enough for want of a Dedication, I am resolv'd it shall not want an Epistle to the Reader too. I shall not take upon me to determine, whether it is lawful to play at Dice or not, leaving that to be disputed betwixt the Fanatick Parsons and the Sharpers; I am sure it is lawful to deal with Dice as with other Epidemic Distempers; and I am confident that the writing a Book about it, will contribute as little towards its Encouragement, as Fluxing and Precipitates do to Whoring.*

It will be to little purpose to tell my Reader, of how great Antiquity the playing at Dice is. I will only let him know, that by the Alcæ Ludus, the Antients comprehended all Games, which were subjected to the determination of mere Chance; this sort of Gaming was strictly forbid by the Emperor Justinian, Cod. Lib. 3. Tit. 43. under very severe Penalties; and Phocius Nomocan. Tit. 9. Cap. 27. acquaints us, that the Use of this was altogether denied the Clergy of that time. Seneca says very well, Alcator quantò in arte est melior, tantò est nequior; That by how much the one is more skilful in Games, by so much he is the more culpable; or we may say of this, as an ingenious Man says of Dancing, That to be extraordinary good at it, is to be excellant in a Fault; therefore I hope no body will imagine I had so mean a Design in this, as to teach the Art of Playing at Dice.

*A great part of this Discourse is a Translation from Mons. Huygen's Treatise, De ratiociniis in ludo Alcæ; one, who in his Improvements of Philosophy, has but one Superior, and I think few or no Equals. The whole I undertook for my own Divertisement, next to the Satisfaction of
some*

P R E F A C E. v

some Friends, who would now and then be wrangling about the Proportions of Hazards in some Cases that are here decided. All it requir'd was a few spare Hours, and but little Work for the Brain; my Design in publishing it, was to make it of more general Use, and perhaps persuade a raw Squire, by it, to keep his Money in his Pocket; and if, upon this account, I should incur the Clamours of the Sharpers, I do not much regard it, since they are a sort of People the World is not bound to provide for.

You will find here a very plain and easy Method of the Calculation of the Hazards of Game, which a man may understand, without knowing the Quadratures of Curves, the Doctrine of Series's, or the Laws of Concentripetation of Bodies, or the Periods of the Satellites of Jupiter; yea, without so much as the Elements of Euclid. There is nothing required for the comprehending the whole, but common Sense and practical Arithmetick; saving a few Touches of Algebra, as in the first Three Propositions, where the Reader, without suspicion of Popery, may make use of a strong implicit Faith; tho' I must confess, it does not much recommend it self to me in these Purposes; for I had rather he
would

would enquire, and I believe he will find the Speculation not unpleasant.

Every man's Success in any Affair is proportional to his Conduct and Fortune. Fortune (in the sense of most People) signifies an Event which depends on Chance, agreeing with my Wish; and Misfortune signifies such an one, whose immediate Causes I don't know, and consequently can neither foretel nor produce it (for it is no Heresy to believe, that Providence suffers ordinary matters to run in the Channel of second Causes). Now I suppose, that all a wise Man can do in such a Case is, to lay his Business on such Events, as have the most powerful second Causes, and this is true both in the great Events of the World, and in ordinary Games. It is impossible for a Die, with such determin'd force and direction, not to fall on such a determin'd side, only I don't know the force and direction which makes it fall on such a determin'd side, and therefore I call that Chance, which is nothing but want of Art; that only which is left to me, is to wager where there are the greatest number of Chances, and consequently the greatest probability to gain; and the whole Art of Gaming, where there is any thing of Hazard, will
be

be reduc'd to this at last, viz. in dubious Cases to calculate on which side there are most Chances; and tho' this can't be done in the midst of Game precisely to an Unit, yet a Man who knows the Principles, may make such a conjecture, as will be a sufficient direction to him; and tho' it is possible, if there are any Chances against him at all, that he may lose, yet when he chuseth the safest side, he may part with his Money with more content (if there can be any at all) in such a Case.

I will not debate, whether one may engage another in a disadvantageous Wager. Games may be suppos'd to be a tryal of Wit as well as Fortune, and every Man, when he enters the Lists with another, unless out of Complaisance, takes it for granted, his Fortune and Judgment, are, at least, equal to those of his Play-Fellow; but this I am sure of, that false Dice, Tricks of Leger-de-main, &c. are inexcusable, for the question in Gaming is not, Who is the best Jugler?

The Reader may here observe the Force of Numbers, which can be successfully applied, even to those things, which one would imagine are subject to no Rules. There are very few things which we know,
which.

which are not capable of being reduc'd to a Mathematical Reasoning; and when they cannot, it's a sign our Knowledge of them is very small and confus'd; and where a mathematical reasoning can be had, it's as great folly to make use of any other, as to grope for a thing in the dark, when you have a Candle standing by you. I believe the Calculation of the Quantity of Probability might be improved to a very useful and pleasant Speculation, and applied to a great many Events which are accidental, besides those of Games; only these Cases would be infinitely more confus'd, as depending on Chances which the most part of Men are ignorant of; and as I have hinted already, all the Politicks in the World, are nothing else but a kind of Analysis of the Quantity of Probability in casual Events, and a good Politician signifies no more, but one who is dextrous at such Calculations; only the Principles which are made use of in the Solution of such Problems, can't be studied in a Closet, but acquir'd by the Observation of Mankind.

*There is likewise a Calculation of the Quantity of Probability founded on Experience, to be made use of in Wagers about any thing; it is odds, if a Woman is
with*

with Child, but it shall be a Boy; and if you would know the just odds, you must consider the Proportion in the Bills that the Males bear to the Females: The Yearly Bills of Mortality are observ'd to bear such Proportion to the live People as 1 to 30, or 26; therefore it is an even Wager, that one out of thirteen, dies within a Year (which may be a good reason, tho' not the true, of that foolish piece of Superstition), because, at this rate, if 1 out of 26 dies, you are no loser. It is but 1 to 18 if you meet a Parson in the Street, that he proves to be a Non-Juror, because there is but 1 of 36 that are such. It is hardly 1 to 10, that a Woman of Twenty Years old has her Maidenhead, and almost the same Wager, that a Town-Spark of that Age has not been clap'd. I think a Man might venture some odds, that 100 of the Gens d'arms beats an equal Number of Dutch Troopers; and that an English Regiment stands its ground as long as another, making Experience our Guide in all these Cases and others of the like nature.

But there are no casual Events, which are so easily subjected to Numbers, as those of Games; and I believe, there the Speculation might be improved so far, as to bring in the Doctrine of the Series's and Logarithms.

richms. Since Gaming is become a Trade, I think it fit the Adventurers should be upon the Square; and therefore in the Contrivance of Games there ought to be a strict Calculation made use of, that they mayn't put one Party in more probability to gain than another; and likewise, if a Man has a considerable Venture; he ought to be allow'd to withdraw his Money when he pleases, paying according to the Circumstances he is then in: and it were easy in most Games to make Tables, by Inspection of which, a Man might know what he was either to pay or receive, in any Circumstances you can imagin, it being convenient to save a part of one's Money, rather than venture the loss of it all.

I shall add no more, but that a Mathematician will easily perceive, it is not put in such a Dress as to be taken notice of by him, there being abundance of Words spent to make the more ordinary sort of People understand it.

FOR the sake of those who are not vers'd in *Mathematicks*, I have added the following Explanation of *Signs*.

$\equiv$ *Equal.*

$+$ *More, or to be added.*

$-$ *Less, or to be subtracted.*

$\times$ *Multiplied.*

$\div$ *Divided.*

EXAMPLE.

$3 \times 4 + 3 - 1 \equiv 14 \equiv \frac{14}{5}a$, is to be read thus ;

3 multiplied in 4, more by 3, less by 1, is equal to 14, which is equal to five ninth parts of *a*.

OR the like of this who are not
 versed in Mathematics, I have added
 the following Explanation of Signs.

- = Equal.
- + Plus, or to be added.
- Minus, or to be subtracted.
- x Multiplied.
- ÷ Divided.

EXAMPLES.

$3 \times 4 + 3 - 1 = 14 = \frac{14}{2}$ is to be
 read thus:

3 multiplied in 4, more by 3, less by 1,
 is equal to 14, which is equal to the
 ninth parts of 14.



An EXACT

METHOD

For SOLVING the

HAZARDS of GAME.

ALTHO' the Events of Games, which Fortune solely governs, are uncertain, yet it may be certainly determin'd, how much one is more ready to lose than gain. For *Example*: If one should wager, at the first throw with one Die, to throw six, it's an accident if he gains or not; but by how much it's more probable he will lose than gain, is really determin'd by the Nature of the thing, and capable of a strict Calculation. So likewise if I should play with another on this condition, that the Victory should be to the three first Games, and I had gain'd one already,

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ready, it is still uncertain who shall gain the third ; yet by a demonstrative Reasoning, I can estimate both the Value of his Expectation and mine, and consequently (if we agree to leave the Game imperfect) determine how great a share of the Stakes belong to me, and how much to my Play-fellow ; or if any were desirous to take my place, at what rate I ought to sell it. Hence may arise innumerable Queries among two, three, or more Gamesters : and since the Calculation of these things is a little out of the common Road, and can be oft-times apply'd to good purpose, I shall briefly here shew how it is to be done, and afterwards explain those things which belong properly to the Dice.

In both Cases I shall make use of this Principle, *One's Hazard or Expectation to gain any thing, is worth so much, as, if he had it, he could purchase the like Hazard or Expectation again in a just and equal Game.*

For *Example*, if one, without my Knowledge, should hide in one hand 7 Shillings, and in his other 3 Shillings, and put it to my choice which Hand I would take, I say this is as much worth to me, as if he should give me 5 Shillings ; because, if I have 5 Shillings, I can purchase as good a Chance again, and that in a fair and just Game.

PROPOSITION I.

If I expect a or b, either of which, with equal probability, may fall to me, then my Expectation is worth $\frac{a+b}{2}$, that is, the half Sum of a and b.

THAT I may not only demonstrate, but likewise investigate this Rule, suppose the Value of my Expectation be x ; by the former Principle having x , I can purchase as good an Expectation again in a fair and just Game. Suppose then I play with another on these terms, That every one stakes x , and the Gainer give to the Loser a , this Game is just, and it appears, that at this rate, I have an equal hazard either to get a if I lose the Game, or $2x - a$ if I gain; for in this case I get $2x$, which are the Stakes, out of which I must pay the other a ; but if $2x - a$ were worth b , then I have an equal hazard to get a or b ; therefore making $2x - a = b$, $x = \frac{a+b}{2}$, which is the Value of my Expectation. The Demonstration is easy; for having $\frac{a+b}{2}$, I can play with another who will stake $\frac{a+b}{2}$ against it, on this condition, that the Gainer should give to the Loser a ;

by this means I have an equal Expectation to get a if I lose, or b if I win; for in the last case I get $a + b$ the Stakes, out of which I must pay a to my Play-fellow.

In *Numbers*: if I had an equal hazard to get 3 or 7, then by this Proposition, my Expectation is worth 5, and it is certain, having 5, I may have the same Chance; for if I play with another, so that every one stakes 5, and the Gainer pay to the Loser 3, this is a fair way of gaming; and it is evident I have an equal hazard to get 3 if I lose, or 7 if I gain.

PROP. II.

If I expect a , b , or c , either of which, with equal facility, may happen, then the Value of my Expectation is $\frac{a+b+c}{3}$, or the third part of the Sum of a , b , and c .

FOR the Investigation of which, suppose x be the value of my Expectation; then x must be such, as I can purchase with it the same Expectation in a just Game: Suppose the Conditions of the Game be, that playing with two others, each of us stakes x , and I bargain with one of the Gamesters, if I win, to give him b , and he shall do the same to me; but with
the

the other, that if I gain, I shall give him c , and *vice versâ*; this is fair play: And here I have an equal hazard to get b , if the first win, c if the second, or $3x - b - c$ if I gain myself; for then I get $3x$, *viz.* the Stakes, of which I give the one b and the other c ; but if $3x - b - c$ be equal to a , I have an equal Expectation of a , b , or c ; therefore making $3x - b - c = a$, $x = \frac{a+b+c}{3}$, which is the Value of my Expectation. After the same Method you will find, if I had an equal hazard to get a , b , c , or d , the Value of my Expectation $\frac{a+b+c+d}{4}$, that is the fourth part of the Sum of a , b , c , and d , &c.



PROP. III.

If the number of Chances, by which a falls to me, be p , and the number of Chances, by which b falls, be q , and supposing all the Chances do happen with equal facility, then the Value of my Expectation is $\frac{pa + bq}{p + q}$; i. e. the Product of a multiplied in the number of its Chances added to the Product of b , multiplied into the number of its Chances, and the Sum divided by the number of Chances both of a and b .

SUPPOSE, as before, x be the Value of my Expectation; then if I have x , I must be able to purchase with it that same Expectation again in a fair Game: For this I shall take as many Play-fellows as, with me, make up the number of $p + q$, of which let every one stake x , so the whole Stake will be $px + qx$, and every one plays with equal hopes of winning; with as many of my Fellow-Gamesters as the Number q stands for, I make this bargain one by one, that whoever of them gains shall give me b , and if I win, I shall do so to them; with every one of the rest of the Gamesters, whose Number is $p - 1$, I make this bargain, that whoever of them

gains,

gains, shall give me a , and I shall give every one of them as much, if I gain : It's evident this is fair play ; for no Man here is injur'd ; and in this case I have q Expectations to gain b , and $p - 1$ Expectations to gain a , and 1 Expectation (*viz.* when I win myself) to get $px + qx - bq - ap + a$; for then I am to deliver b to every one of the q Players, and a to every one of the $p - 1$ Gamesters, which makes $qb + pa - a$; if therefore $qx + bx - bq - ap + a$ were equal to a , I would have p Expectations of a (since just now I had $p - 1$ Expectations of it) and q Expectations of b , and so would have just come to my first Expectation ; therefore putting $px + qx - bq - ap + a = a$, then is $x = \frac{ap + bq}{p + q}$.

In *Numbers* : If I had 3 Chances to gain for 13, and 2 for 8, by this Rule, my hazard is worth 11 ; for 13 multiplied by 3 gives 39, and 8 by 2 16, these two added, make 55, divided by 5 is 11 ; and I can easily shew, if I have 11, I can come to the like Expectation again ; for playing with four others, and every one of us staking 11, with two of them I make this bargain, that whoever gains shall give me 8, and I shall too do so to them ; with the other two I make this bargain, that whoever

ever gains shall give me 13, and I them as much if I gain : it appears, by this means I have two Expectations to get 8, *viz.* if any of the first two gain, and 3 Expectations to get 13, *viz.* if either I or any of the other two gain ; for in this case I gain the Stakes, which are 55, out of which I am oblig'd to give the first two 8, and the other two 13, and so there remains 13 for myself.

PROP. IV.

That I may come to the Question propos'd, viz. The making a just Distribution amongst Gamesters, when their Hazards are unequal ; we must begin with the most easy Cases.

SUPPOSE then I play with another, on condition that he who wins the three first Games shall have the Stakes, and that I have already gain'd two, I would know, if we agree to break off the Game, and part the Stakes justly, how much falls to my share ?

The first thing we must consider in such Questions is the number of Games that are wanting to both : For *Example*, if it had been agreed betwixt us, that he should have the Stakes who gain'd the first 20 Games,

HAZARDS of Game.

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Games, and if I had gain'd already 19, and my Fellow-Gamester but 18, my hazard is as much better than his in that case, as in this propos'd, *viz.* When of 3 Games I have 2, and he but one, because in both cases there's 2 wanting to him, and 1 to me.

In the next place, to find the portion of the Stakes due to each of us, we must consider what would happen if the Game went on; it is certain, if I gain the first Game, I get the Stake, which I call a ; but if he gain'd, both our Lots would be equal, and so there would fall to each of us $\frac{1}{2}a$; but since I have an equal hazard to gain or lose the first Game, I have an equal Expectation to gain a , or $\frac{1}{2}a$, which, by the first *Proposition*, is as much worth as the half Sum of both, *i. e.* $\frac{3}{4}a$, so there is left to my Fellow-Gamester $\frac{1}{4}a$; from whence it follows, that he who would buy my Game, ought to pay me for it $\frac{3}{4}a$; and therefore, he who undertakes to gain one Game before another gains two, may wager 3 to 1.



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PROP.

PROP. V.

Suppose I want but one Game, and my Fellow-Gamester three, it is required to make a just Distribution of the Stake.

LET us here likewise consider in what state we should be, if I or he gain'd the first Game; if I gain, I have the Stake a , if he, then he wants yet 2 Games, and I but 1, and therefore we should be in the same Condition which is supposed in the former Proposition; and so there would fall to my Share, as was demonstrated there, $\frac{3}{4}a$; therefore with equal facility there may happen to me a , or $\frac{3}{4}a$, which, by the first Proposition, is worth $\frac{7}{8}a$, and to my Fellow-Gamester there is left $\frac{1}{8}a$, and therefore my hazard to his is as 7 to 1.

As the Calculation of the former Proposition was requisite for this, so this will serve for the following. If I should suppose myself to want but one Game, and my Fellow four, (by the same Method) you will find $\frac{15}{16}$ of the Stake belongs to me, and $\frac{1}{16}$ to him.

PROP. VI.

Suppose I want two Games, and my Fellow-Gamester three.

THEN by the next Game it will happen that I want but one, and he three, which (by the preceding Proposition) is worth $\frac{7}{8}a$; or that we should both want two, whence there will be $\frac{1}{2}a$ due to each of us: now I being in an equal probability to gain or lose the next Game, I have an equal hazard to gain $\frac{7}{8}a$ or $\frac{1}{2}a$, which by the first Proposition is worth $\frac{1}{16}a$; and so there are eleven parts of the Stakes due to me, and five to my Fellow.

PROP. VII.

Let us suppose I want two Games, and my Fellow four.

IF I gain the next Game, then I shall want but one, and my Fellow four; but if I lose it, then I shall want two, and he three: So I have an equal hazard for gaining $\frac{1}{16}a$, or $\frac{1}{16}a$, which, by the first, is worth $\frac{1}{16}a$: So it appears, that he who is to gain two Games for the other's four, is in a better condition than he who is to

gain one for the other's two; for my share in the first case is $\frac{3}{4}a$ or $\frac{1}{4}\frac{3}{2}a$, which is less than $\frac{1}{2}\frac{3}{2}$, my share in the last.

PROP. VIII.

Let us suppose three Gamesters, whereof the first and second want 1 Game, but the third 2.

TO find the share of the first, we must consider what would happen if either he, or any of the other two gain'd the first Game; if he gains, then he has the Stake a ; if the second gain, he has nothing; but if the third gain, then each of them would want a Game, and so $\frac{1}{3}a$ would be due to every one of them. Thus the first Gamester has one Expectation to gain a , one to gain nothing, and one for $\frac{1}{3}a$, (since all are in equal probability to gain the first Game) which by the second Proposition is worth $\frac{4}{9}a$: Now since the second Gamester's Condition is as good, his Share is likewise $\frac{4}{9}a$, and so there remains to the third $\frac{1}{9}a$, whose Share might have been as easily found by itself.



PROP. IX.

In any number of Gamesters you please, amongst whom there are some who want more, some fewer Games: To find what is any one's share in the Stake, we must consider what would be due to him, whose Share we investigate, if either he, or any of his Fellow-Gamesters should gain the next following Game; add all their Shares together, and divide the Sum by the number of the Gamesters, the Quotient is his Share you were seeking.

SUPPOSE three Gamesters *A*, *B*, and *C*; *A* wants 1 Game, *B* 2, and *C* likewise 2, I would find what is the share of the Stake due to *B*, which I shall call *q*.

First, we must consider what would fall to *B*'s Share, if either he, *A*, or *C*, wins the next Game; if *A* wins, the Game is ended, so he gets nothing; if *B* himself gain, then he wants 1 Game, *A* 1, and *C* 2; therefore, by the former Proposition, there is due to him in that case $\frac{4}{9}q$, then if *C* gains the next play, then *A* and *C* would want but 1, and *B* 2; and therefore, by the eighth Proposition, his Share would be worth $\frac{1}{9}q$; add together what is due to *B* in all these three Cases, viz. $0\frac{4}{9}q$, $\frac{1}{9}q$.

$\frac{1}{9}q$, the Sum is $\frac{5}{9}q$, which being divided by 3, the number of Gamesters, gives $\frac{5}{27}q$, which is the Share of *B* sought for: The Demonstration of this is clear from the second Proposition, because *B* has an equal hazard to gain $0\frac{4}{9}q$ or $\frac{1}{9}q$, that is $\frac{0 + \frac{4}{9}q + \frac{1}{9}q}{3}$, i. e. $\frac{5}{27}q$: now it's evident the Divisor 3 is the number of the Gamesters.

To find what is due to one in any case; viz. if either he, or any of his Fellow-Gamesters win the following Game; we must consider first the more simple Cases, and by their help the following; for as this Case could not be solv'd before the Case of the eighth Proposition was calculated, in which, the Games wanting were 1, 1, 2; so the Case, where the Games wanting are 1, 2, 3, cannot be calculated, without the Calculation of the Case, where the Games wanting are 1, 2, 2, (which we have just now perform'd) and likewise of the Case, where the Games wanting are 1, 1, 3, which can be done by the eighth: And by this means you may reckon all the Cases comprehended in the following Tables, and an infinite number of others.

Games

Games wanting	1, 1, 2.	1, 2, 2.	1, 1, 3.	1, 2, 3.
Their Shares	4, 4, 1.	17, 5, 5.	13, 13, 1.	19, 6, 2.
	9	27	27	27

Games wanting	1, 1, 4	1, 1, 5	1, 2, 4	1, 2, 5
Shares	40, 40, 1	121, 121, 1	178, 58, 7	542, 179, 8
	81	243	243	729

Games wanting	1, 3, 3	1, 3, 4	1, 3, 5
Their Shares	65, 8, 8	616, 82, 31	629, 87, 13
	81	729	729

Games wanting	2, 2, 3	2, 2, 4	2, 2, 5
Their Shares	34, 34, 13	338, 338, 53	353, 353, 23
	81	729	729

Games wanting	2, 3, 3	2, 3, 4	2, 3, 5
Their Shares	133, 55, 55	451, 195, 83	433, 635, 119
	243	729	1187

As for the *Dice*; these Questions may be proposed, at how many Throws one may wager to throw 6, or any Number below that, with one Die; How many Throws are required for 12 upon two Dice; or 18 on 3; and several other Questions to this purpose.

For the resolving of which, it must be consider'd, that in one Die there are six different Throws, all equally probable to come up; for I suppose the Die has the exact

exact figure of a Cube: On two Dice there are 36 different throws; for in respect to every throw of one Die, any one throw of the 6 of the other Die may come up; and 6 times 6 make 36. In three Dice there are 216 different throws; for in relation to any of the 36 throws of two Dice, any one of the six of the third may come up; and 6 times 36 make 216: So in four Dice there are 6 times 216 throws, that is, 1296: And so forward you may reckon the throws of any number of Dice, taking always, for the Addition of a new Die, 6 times the number of the preceding.

Besides, it must be observ'd, that in two Dice there is only one way 2 or 12 can come up; two ways that 3 or 11 can come up; for if I shall call the Dice A and B, to make 3 there may be 1 in A, and 2 in B, or 2 in A, and 1 in B; so to make 11, there may be 5 in A, and 6 in B, or 6 in A, and 5 in B; for 4 there are three Chances, 3 in A, and 1 in B, 3 in B, and 1 in A, or 2 as well in A as B; for 10 there are likewise three Chances; for 5 or 9 there are four Chances; for 6 or 8 five Chances; for 7 there are six Chances.

In 3 Dice there are found for	{	3 or 18	1
		4 or 17	3
		5 or 16	6
		6 or 15	10
		7 or 14	15
		8 or 13	21
		9 or 12	25
	[	10 or 11	27

PROP. X.

To find at how many times one may undertake to throw 6 with one Die.

IF any should undertake to throw 6 the first time, it's evident, there's one Chance gives him the Stake, and five which give him nothing; for there are 5 throws against him, and only one for him. Let the Stake be call'd a , then he has one Expectation to gain a , and five to gain nothing, which, by the third Proposition, is worth $\frac{1}{6}a$, and there remains for the other $\frac{5}{6}a$; so he who undertakes, with one Die, to throw 6 the first time, ought to wager only 1 to 5.

2. Suppose one undertake, at two Throws of 1 Die, to throw 6, his Hazard is calculated thus; if he throw 6 at the first, he has a the Stake; if he do not, there re-

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mains to him one throw, which, by the former Case, is worth $\frac{1}{6}a$; but there is but one Chance which gives him 6 at the first throw, and five Chances against him; so there is one Chance which gives him a , and five which give him $\frac{1}{6}a$, which by the second Proposition, is worth $\frac{1}{3}\frac{1}{6}a$, so there remains to his Fellow-Gamester $\frac{2}{3}\frac{5}{6}a$; so the Value of my Expectation to his, is as 11 to 25, *i. e.* less than 1 to 2.

By the same method of Calculation, you will find, that his hazard who undertakes to throw 6 at three times with one Die, is $\frac{2}{3}\frac{1}{6}a$; so that he can only lay 91 against 125, which is something less than 3 to 4.

He who undertakes to do it at four times, his hazard is $\frac{6}{1}\frac{1}{6}\frac{1}{6}a$, so he may wager 671 against 625, that is, something more than 1 to 1.

He who undertakes to do it at five times, his hazard is $\frac{4}{7}\frac{6}{7}\frac{5}{7}\frac{1}{6}a$, so he can wager 4651 against 3125, that is, something less than 3 to 2.

His hazard who undertakes to do it at 6 times, is $\frac{3}{4}\frac{1}{6}\frac{5}{6}\frac{3}{4}\frac{1}{6}a$, and he can wager 31031 against 15625, that is, something less than 2 to 1.

Thus any number of throws may be easily found; but the following Proposition will shew you a more compendious way of Calculation.

PROP.

PROP. XI.

To find at how many times one may undertake to throw 12 with two Dice.

IF one should undertake it at one throw, it's clear he has but one Chance to get the Stake a , and 35 to get nothing; which, by the third Proposition, is worth $\frac{1}{36}a$.

He who undertakes to do it at twice, if he throw 12 the first time, gains a ; if otherwise, then there remains to him one throw, which, by the former Case, is worth $\frac{1}{36}a$; but there is but one Chance which gives 12 at the first throw, and 35 Chances against him; so he has 1 Chance for a , and 35 for $\frac{1}{36}a$, which by the third Proposition is worth $\frac{71}{1296}a$, and there remains to his Fellow-Gamester $\frac{1225}{1296}a$.

From these it's easy to find the Value of his hazard, who undertakes it at four times, passing by his case who undertakes it at three times.

If he who undertakes to do it at four times throws 12 the first or second Cast, then he has a ; if not, there remains two other throws, which, by the former Case, are worth $\frac{71}{1296}a$; but for the same reason, in his two first throws, he has 71 Chances which give him a , against 1225 Chances,

in which it may happen otherwise ; therefore at first he has 71 Chances which give him a , and 1225 which give him $\frac{7\frac{1}{2}}{1296}a$, which by the third Proposition is worth $\frac{178999}{1679616}a$, which shews that their hazards to one another are as 178991 to 1500625.

From which Cases it is easy to find the Value of his Expectation, who undertakes to do it at 8 times, and from that, his Case who undertakes to do it at 16 times ; and from his Case who undertakes to do it at 8 times, and his likewise who undertakes to do it at 16 times ; it is easy to determine his Expectation who undertakes it at 24 times : In which Operation, because that which is principally sought, is the number of throws, which makes the hazard equal on both sides, *viz.* to him who undertakes, and he who offers, you may, without any sensible Error, from the Numbers (which else would grow very great) cut off some of the last Figures. And so I find, that he who undertakes to throw 12 with two Dice, at 24 times, has some loss ; and he who undertakes it at 25 times, has some advantage.



PROP.

PROP. XII.

To find with how many Dice one can undertake to throw two Sixes at the first Cast.

THIS is as much, as if one would know, at how many throws of one Die, he may undertake to throw twice six: now if any should undertake it at two throws, by what we have shewn before, his hazard would be $\frac{1}{36}a$; he who would undertake to do it at 3 times, if his first throw were not 6, then there would remain two throws, each of which must be 6, which (as we have said) is worth $\frac{1}{36}a$; but if the first throw be 6, he wants only one 6 in the two following throws, which by the tenth Proposition, is worth $\frac{1}{36}a$: but since he has but one Chance to get 6 the first throw, and five to miss it; he has therefore, at first, one Chance for $\frac{1}{36}a$, and five Chances for $\frac{1}{36}a$, which, by the third Proposition, is worth $\frac{1}{2} \cdot \frac{6}{36}a$, or $\frac{1}{12}a$; after this manner still assuming 1 Chance more, you will find that you may undertake to throw two Sixes at 10 throws of one Die, or 1 throw of ten Dice, and that with some advantage.

PROP.

PROP. XIII.

If I am to play with another one Throw, on this condition, that if 7 comes up I gain, if 10 be gains; if it happens that we must divide the Stake, and not play, to find how much belongs to me, and how much to him.

BEcause of the 36 different Throws of the two Dice, there are six which give 7, and 3 which give 10, and 27 which equals the Game, in which case there is due to each of us $\frac{1}{2}a$: But if none of the 27 should happen, I have 6, by which I may gain a , and 3, by which I may get nothing, which by the third Proposition, is worth $\frac{2}{3}a$; so I have 27 Chances for $\frac{1}{2}a$, and 9 for $\frac{2}{3}a$, which, by the third Proposition, is worth $\frac{1}{2}\frac{3}{4}a$, and there remains to my Fellow-Gamester $\frac{1}{2}\frac{1}{3}a$.



PROP. XIV.

If I were playing with another by turns, with two Dice, on this condition, that if I throw 7 I gain, and if he throw 6 he gains, allowing him the first throw: To find the proportion of my Hazard to his.

SUPPOSE I call the Value of my Hazard x , and the Stakes a , then his Hazard will be $a - x$; then whenever it's his turn to throw, my Hazard is x , but when it's mine, the Value of my Hazard is greater. Suppose I then call it y ; now because of the 36 throws of two Dice, there are five which give my Fellow-Gamester 6, thirty-one which bring it again to my turn to throw, I have five Chances for nothing, and thirty-one for y , which, by the third Proposition, is worth $\frac{31}{36}y$; but I suppos'd at first my Hazard to be x ; therefore $\frac{31}{36}y = x$, and consequently $y = \frac{36}{31}x$. I suppos'd likewise, when it was my turn to throw, the Value of my Hazard was y ; but then I have six Chances which give me 7, and consequently the Stake, and thirty which give my Fellow the Dice, that is, make my Hazard worth x ; so I have six Chances for a , and thirty for x ,
which,

which, by Proposit. 3. is worth $\frac{6a + 30x}{36}$; but this by supposition is equal to y , which is equal (by what has been prov'd already) to $\frac{36}{31}x$; therefore $\frac{30x + 6a}{36} = \frac{36}{31}x$, and consequently $x = \frac{3}{8}a$, the Value of my Hazard, and that of my Fellow-Gamester is $\frac{3}{8}a$, so that mine is to his as 31 to 30.

Here follow some Questions which serve to exercise the former Rules.

1. *A* and *B* play together with two Dice, *A* wins if he throws 6, and *B* if he throws 7 ; *A* at first gets one throw, then *B* two, then *A* two, and so on by turns, till one of them wins. I require the proportion of *A*'s Hazard to *B*'s? *Answer*, It is as 10355 to 12276.

2. Three Gamesters, *A*, *B*, and *C*, take 12 Counters, of which there are four white and eight black ; the Law of the Game is this, that he shall win, who, hood-wink'd, shall first chuse a white Counter ; and that *A* shall have the first choice, *B* the second, and *C* the third, and so, by turns, till one of them win. *Quær*. What is the proportion of their Hazards?

3. *A* wagers with *B*, that of 40 Cards, that is, 10 of every Suit, he will pick out four, so that there shall be one of every suit. *A*'s Hazard to *B*'s in this case is as 1000 to 8139.

4. Supposing, as before, 4 white Counters and 8 black, *A* wagers with *B*, that out of them he shall pick 7 Counters, of which there are 3 white. I require the proportion of *A*'s Hazard to *B*'s?

5. *A* and *B* taking 12 Counters, each play with three Dice after this manner, that if 11 comes up, *A* shall give one Counter to *B*; but if 14 comes up, *B* shall give one to *A*, and that he shall gain who first has all the Counters. *A*'s Hazard to *B*'s is 244140625 to 282429536481.

The *Calculus* of the preceding Problems is left out by *Monf. Huygens*, on purpose that the ingenious Reader may have the satisfaction of applying the former Method himself; it is in most of them more laborious than difficult: for *Example*, I have pitch'd upon the second and third, because the rest can be solv'd after the same Method.

PROBLEM I.

The first Problem is solv'd by the Method of Prop. 14. only with this difference, that after you have found the Share due to *B*, if *A* were to get no first throw, you must subtract from it $\frac{5}{8}$ of the Stake which is due to *A* for his Hazard of throwing six at the first throw.

PROBLEM 2.

As for the second Problem, it is solv'd thus; Suppose *A*'s Hazard, when it is his own turn to chuse, be *x*, when it is *B*'s, be *y*, and when it is *C*'s, be *z*; it is evident, when out of 12 Counters, of which there are 4 white and 8 black, he endeavours to chuse a white one, he has four Chances to get it, and eight to miss it; that is, he has four Chances to get the Stake *a*, and eight to make his hazard worth *y*: so $x = \frac{4a + 8y}{12}$, and consequently $y = \frac{12x - 4a}{8}$.

When it is *B*'s turn to chuse, then he has four Chances for nothing, and eight for *z*, (that is, to bring it to *C*'s turn) consequently $y = \frac{8z}{12} = \frac{12x - 4a}{8}$; this Equation reduc'd gives $z = \frac{9x - 3a}{4}$; when it comes

comes to *C*'s turn to chuse, then *A* has four Chances for nothing, and eight for x , consequently $x = \frac{8}{12}x$, therefore $\frac{8}{12}x = \frac{9x-3a}{4}$; this equation reduc'd gives $x = \frac{2}{9}a$, and consequently there remains to *B* and *C* $\frac{10}{9}a$, which must be shar'd after the same manner, that is, so that *B* have the first choice, *C* the next, and so on, till one of them gain; the reason is, because it had been just in *A* to have demanded $\frac{2}{9}$ of the Stake for not playing, and then the Seniority fell to *B*; now $\frac{10}{9}a$, parted betwixt *B* and *C*, by the former Method, gives $\frac{6}{9}$ to *B*, and $\frac{4}{9}$ to *C*; so *A*, *B*, and *C*'s Hazards from the beginning were as 9, 6, 4.

I have suppos'd here the Sense of the Problem to be, that when any one chus'd a Counter, he did not diminish their number; but if he mis'd of a white one, put it in again, and left an equal hazard to him who had the following choice; for if it be otherwise suppos'd, *A*'s share will be $\frac{5}{12}$, which is less than $\frac{2}{9}$.

Prob. 2. It is evident, that wagering to pick out 4 Cards out of 40, so that there be one of every Suit, is no more than wagering, out of 39 Cards to take 3 which shall be of three proposed Suits; for it is all one which Card you draw first, all the

hazard being, whether out of the 39 remaining you take 3, of which none shall be of the Suit you first drew: Suppose then you had gone right for three times, and were to draw your last Card, it is clear that there are 27 Cards, (*viz.* of the Suits you have drawn before) of which, if you draw any you lose, and 10, of which if you draw any, you have the Stake a ; so you have 10 Chances for a , and 27 for nothing, which, by Prop. 3. is worth $\frac{10}{37}a$. Suppose again you had gone right only for two Draughts, then you have 18 Cards (of the Suits you have drawn before) which make you lose, and 20, which put you in the Case suppos'd formerly, *viz.* where you have but one Card to draw, which, as we have already calculated, is worth $\frac{10}{37}a$; so you have 18 Chances for nothing, and 20 for $\frac{10}{37}a$, which, by Prop. 3. is worth $\frac{10}{703}a$. Suppose again you have 3 Cards to draw, then you have 9 (of the Suit you drew first) which make you lose, and 30 which put you in the case suppos'd last; so you have 9 Chances for nothing, and 30 for $\frac{10}{703}a$, which by Propos. 3. is worth $\frac{30}{2709}a$, or $\frac{10}{901}a$, and you leave to your Fellow-Gamster $\frac{8139}{901}a$; so your hazard is to his as 1000 to 8139.

It is easy to apply this Method to the Games that are in use amongst us: For

Example,

Example, If *A* and *B*, playing at *Backgammon*, *A* had already gain'd one end of three, and *B* none, and if *A* had the Dice in his Hand for the last throw of the second end, all his Men but two upon the Ace Point being already cast off: *Quær*. What is the proportion of *A*'s Hazard to *B*'s?

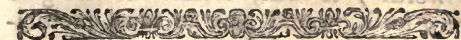
Solution: There being of the 36 Throws of two Dice, six which give Doublets; if *A* throw any of the six, he has the stake *a*; if he throw any of the other thirty, then he wants but one Game, and his Fellow-Gamester three, which, by Prop. V. is $\frac{7}{8}a$; so *A* has six Chances for *a*, and thirty for $\frac{7}{8}a$, which, by Prop. III. is worth $\frac{1 \cdot 2 \cdot 2}{1 \cdot 4 \cdot 4}a$, and there remains to his Play-Fellow $\frac{1 \cdot 5}{1 \cdot 4 \cdot 4}a$; so *A*'s Hazard to *B*'s is as 129 to 15, that is, less than 9 to 1.

Supposing the same Case, and if their Bargain had been, that he who gain'd three ends before the other gain'd one, should have double of what each stak'd, that is, the Stake and a half more, then there had been due to *A* $\frac{2 \cdot 8 \cdot 2}{1 \cdot 4 \cdot 4}$ of the Stake, that is, *B* ought only to take $\frac{1}{4 \cdot 8}$, and leave the rest to *A*.

Thus likewise, if you apply the former Rule to the *Royal-Oak Lottery*, you will find, that he who wagers that any Figure shall come up at the first throw, ought to wager 1 against 31; that he who wagers
it

it shall come up at one of two throws, ought to wager 63 against 961; that he who wagers that a Figure shall come up at once in three times, ought to lay 125055 against 923521, &c. it being only somewhat tedious to calculate the rest. Where you will find, that the equality will not fall as some imagine on 16 Throws, no more than the equality of wagering at how many Throws of one Die 6 shall come up, falls on three; the contrary of which you have seen already demonstrated: you will find by calculation, that he has the Disadvantage, who wagers, that 1 of the 32 different Throws of the *Royal-Oak Lottery* shall come at once of 20 times, and that he has some advantage, who wagers on 22 times, so the nearest to Equality is on 21 times. But it must be remembered, that I have suppos'd in the former Calculation, the Ball in the *Royal-Oak Lottery* to be regular, tho' it can never be exactly so; for he, who has the smallest Skill in Geometry, knows, that there can be no regular Body of 32 sides; and yet this can be of no Advantage to him who keeps it.





*To find the Value of the Throws of
Dice, as to the Quantity.*

Nothing is more easy, than by the former Method to determine the Value of any number of Throws of any number of Dice; for in one throw of a Die, I have an equal Chance for 1, 2, 3, 4, 5, 6, consequently my Hazard is worth their Sum 21 divided by their Number 6, that is, $3\frac{1}{2}$. Now if one throw of a Die be worth $3\frac{1}{2}$, then two throws of a Die, or one throw of two Dice is worth 7, two throws of two Dice, or one throw of four Dice is worth 14, &c. The general Rule being to multiply the Number of Dice, the Number of Throws, and $3\frac{1}{2}$ continually.

This is not to be understood as if it were an equal wager to throw 7, or above it, with two Dice at one throw; for he who undertakes to do so, has the Advantage by 21 against 15. The meaning is only, if I were to have a Guinea, a Shilling, or any thing else, for every Point that I threw with

with two Dice at one throw, my Hazard is worth 7 of these, because he who gave me 7 for it, would have an equal probability of gaining or losing by it, the Chances of the Throws above 7, being as many as of these below it: So it is more than an equal wager to throw 14 at least at two Throws of two Dice, because it is more probable that 14 will come, than any one number besides, and as probable that it will be above it as below it; but if one were to buy this Hazard at the rate above-mention'd, he ought just to give 14 for it. The equal wager in one Throw of two Dice, is to throw 7 at least one time, and 8 at least another time, and so *per vices*: The reason is, because in the first Case I have 21 Chances against 15, and in the second 15 Chances against 21.



of



Of RAFFLING.

IN *Raffling*, the different Throws and their Chances are these; Where it is to be observed, that of the 216 different Throws of three Dice, there are only 96 that give Doubles, or two, at least, of a kind; so it is 4 to 5 that with three Dice you shall throw Doubles, and it is 1 to 35 that you throw a Raffle, or all three of a kind. It is evident likewise, that it is an even wager to throw

Throws. Chan.

3	18	1
4	17	3
5	16	6
6	15	4
7	14	9
8	13	9
9	12	7
10	11	9

11 or above it, because there are as many Chances for 11, and the Throws above it, as for the Throws below it; but tho' it be an even wager to throw 11 at one Throw, it is a disadvantage to wager to throw 22 at two Throws, and far more to wager to throw 33 at three Throws; and yet it is more than an equal Wager that you shall throw 21 at two Throws in *Raffling*, because it is as probable that you will, as that you will not throw 11, at least, the first time, and

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more

more than probable that you will throw 10, at least, the second time.

For an instance of the plainness of the preceding Method, I will shew, how by simple Subtraction, the most part of the former Problems may be solv'd.

Suppose *A* and *B*, playing together, each of them stakes 32 Shillings, and that *A* wants one Game of the number agreed on, and *B* wants two; to find the share of the Stakes due to each of them. It's plain, if *A* wins the next Game, he has the whole 64 Shillings; if *B* wins it, then their Shares are equal; therefore says *A* to *B*, If you will break off the Game, give me 32, which I am sure of, whether I win or lose the next Game; and since you will not venture for the other 32, let us part them equally, that is, give me 16, which, with the former 32 make 48, leaving 16 to you.

Suppose *A* wanted one Game, and *B* three; if *A* wins the next Game, he has the 64 Shillings; if *B* wins it, then they are in the condition formerly suppos'd, in which case there is 48 due to *A*; therefore says *A* to *B*, give me the 48 which I am sure of, whether I win or lose the next Game; and since you will not hazard for the other 16, let us part them equally, that is, give me 8, which, with the former 48, make

make 56, leaving 8 to you; and so all the other Cases may be solv'd after the same manner.

Suppose *A* wagers with *B*, that with one Die he shall throw 6 at one of three Throws, and that each of them stakes 108 Guineas; to find what is the proportion of their Hazards. Now there being in one Throw of a Die but one Chance for 6, and five Chances against it, one Throw for 6 is worth $\frac{1}{6}$ of the Stake; therefore says *B* to *A*, of the 216 Guineas take a sixth part for your first Throw, that is, 36; for your next Throw take a sixth part of the remaining 180, that is, 30; and for your third Throw, take a sixth part of the remaining 150, that is, 25, which in all make 91, leaving to me 125; so his hazard who undertakes to throw 6 at one of three Throws, is 91 to 125.

Suppose *A* had undertaken to throw 6 with one Die at one Throw of four, and that the whole Stake is 1296; says *A* to *B*, every Throw for 6 of one Die, is worth the sixth part of what I throw for; therefore for my first Throw give me 216, which is the sixth part of 1296, and there remains 1080, I must have the sixth part of that, viz. 180, for my second Throw; and the sixth part of the remaining 900, which is 150, for my third Throw; and

the sixth part of the last remainder 750, which is 125 for my fourth; all this added together makes 671, and there remains to you 625; so it is evident, that *A*'s Hazard, in this Case, is to *B*'s 671 to 625.

Suppose *A* is to win the Stakes (which we shall suppose to be 36) if he throws 7 at once or twice with two Dice, and *B* is to have them if he does not; says *B* to *A*, the Chances which give 7 are 6 of the 36, which is as much as 1 of 6; therefore for your first Throw you shall have a sixth part of the 36, which is 6; and for your next Throw a sixth part of the remainder 30, which is 5; this in all makes 11; so you leave 25 to me; so *A*'s Hazard is to *B*'s as 11 to 25.

It were easy, at this rate, to calculate the most intricate Hazards, were it not that Fractions will occur; which, if they be more than $\frac{1}{2}$, may be suppos'd equal to an Unit, without causing any remarkable Error in great Numbers.

It will not be amiss, before I conclude, to give you a Rule for finding in any number of Games the Value of the first, because *Huygens's* Method, in that case, is something tedious.

Suppose *A* and *B* had agreed, that he should have the Stakes who did win the first 9 Games, and *A* had already won one of

of the 9; I would know what share of *B*'s Money is due to *A* for the Advantage of this Game. To find this, take the first eight even Numbers 2, 4, 6, 8, 10, 12, 14, 16, and multiply them continually, that is, the first by the second, the product by the third, &c. take the first eight odd Numbers 1, 3, 5, 7, 9, 11, 13, 15, and do just so by them, the product of the even Number is the Denominator, and the product of the odd Number the Numerator of a Fraction, which expresseth the quantity of *B*'s Money due to *A* upon the winning of the first Game of 9; that is, if each stak'd a number of Guineas, or Shillings, &c. express'd by the product of the even Numbers, there would belong to *A*, of *B*'s Money, the Number express'd by the product of the odd Numbers. For *Example*, Suppose *A* had gain'd one Game of 4, then by this Rule, I take the three first even Numbers, 2, 4, 6, and multiply them continually, which make 48, and the first three odd Numbers, 1, 3, 5, and multiply them continually, which make 15; so there belongs to *A* $\frac{15}{48}$ of *B*'s Money, that is, if each stak'd 48, there would belong to *A*, besides his own, 15 of *B*'s. Now by *Huygens's* Method, if *A* wants but three Games while *B* wants four, there is due to *A* $\frac{3}{5}$ of the Stake; by this Rule there is due

to

to $A \frac{1}{4} \frac{5}{8}$ of B 's Money, which is $\frac{1}{4} \frac{5}{9}$ of the Stake, which, with his own $\frac{4}{9} \frac{8}{8}$ of the Stake, makes $\frac{6}{9} \frac{1}{2}$ or $\frac{1}{3} \frac{1}{2}$ of the Stake; and so in every Case you will find *Huygens's* Method, and this will give you the same Number: A Demonstration of it you may see in a Letter of *Monfieur Pascal's* to *Monfieur Fermat*; though it be otherwise express'd there than here, yet the consequence is easily supply'd. To prevent the labour of Calculation, I have subjoin'd the following Table, which is calculated for two Gamesters, as *Monfieur Huygens's* is for three.

If each of us stake 256 Guineas in

There belongs to me 256 of my
Play-fellow

	6	5	4	3	2	1
1 st Game	63	70	80	96	128	256
2 1 st Games	126	140	160	192	256	
3 1 st Games	182	200	224	256		
4 1 st Games	224	240	256			
5 1 st Games	248	256				
6 1 st Games	256					

The Use of the Table is plain; for let our Stakes be what they will, I can find the Portion due to me upon the winning the first, or the first two Games, &c. of 2, 3, 4, 5, 6. For *Example*, If each of us had stak'd 4 Guineas, and the number of Games to be play'd were 3, of which I had gain'd 1, say, As 256 is to 96, so is 4 to a fourth.

$$256 : 96 :: 4 : 1\frac{1}{4}.$$

To find what is the Value of his Hazard, who undertakes, at the first Throw, to cast Doublets, in any given number of Dice.

In two Dice it is plain, to avoid Doublets, every one of the six different Throws of the first, can only be combin'd with five of the second, because one of the six is of the same kind, and consequently makes Doublets; for the same reason, the thirty Throws of two Dice, which are not Doublets, can only be combin'd with four Throws of a third Dice, and three Throws of a fourth Dice; so generally it is this Series,

$$\frac{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 0, \text{ \&c.}}{6 \times 6 \times 6 \times 6 \times 6 \times 6, \text{ \&c.}}$$

The second Series is the Sum of the Chances, and the first the Number of Chances against him who undertakes to throw Doublets, each Series to be continu'd so many terms, as are the number of Dice. For *Example*, If one should undertake to throw Doublets at the first Throw of four Dice, his Adversary's Hazard is $\frac{6 \times 5 \times 4 \times 3}{6 \times 6 \times 6 \times 6} = \frac{360}{1296}$ or $\frac{5}{18}$, leaving to him $\frac{13}{18}$, so he has 13 to 5. In seven Dice, you see the Chances against him are 0, because then there must necessarily be Doublets.



of

0 x 2 x 4 x 6 x 8 x 10 x 12 x 14 x 16 x 18 x 20 x 22 x 24 x 26 x 28 x 30 x 32 x 34 x 36 x 38 x 40 x 42 x 44 x 46 x 48 x 50 x 52 x 54 x 56 x 58 x 60 x 62 x 64 x 66 x 68 x 70 x 72 x 74 x 76 x 78 x 80 x 82 x 84 x 86 x 88 x 90 x 92 x 94 x 96 x 98 x 100



Of WHIST.

If there be four playing at Whist, it is 15 to 1 that any two of them shall not have the four Honours, which I demonstrate thus.

Suppose the four Gamesters be *A*, *B*, *C*, *D*: If *A* and *B* had, while the Cards are a dealing, already got three Honours, and wanted only one, since it is as probable that *C* and *D* will have the next Honour, as *A* and *B*; if *A* and *B* had laid a Wager to have it, there is due to them but $\frac{1}{2}$ of the Stake: If *A* and *B* wanted two of the four, and had wager'd to have both those two, then they have an equal hazard to get nothing, if they miss the first of these two, or to put themselves in the former Case if they get; so they have an equal Hazard to get nothing, or $\frac{1}{2}$, which, by *Prop. 1.* is worth $\frac{1}{4}$ of the Stake; so if they want three Honours, you will find due to them $\frac{1}{4}$ of the Stake; and if they wanted four, $\frac{1}{16}$ of the Stake, leaving *C* and *D* $\frac{15}{16}$; so *C* and *D* can wager 15 to 1, that

A and *B* shall not have all the four Honours.

It is 11 to 5 that A and B shall not have three of the four Honours, which I prove thus :

It is an even Wager, if there were but three Honours, that *A* and *B* shall have two of these three, since 'tis as probable that they will have two of the three, as that *C* and *D* shall have them ; consequently, if *A* and *B* had laid a Wager to have two of three, there is due to them $\frac{1}{2}$ of the Stake. Now suppose *A* and *B* had wager'd to have three of four, they have an equal hazard to get the first of the four, or miss it ; if they get it, then they want two of the three, and consequently there is due to them $\frac{1}{2}$ of the Stake ; if they miss it, then they want three of the three, and consequently there is due to them $\frac{1}{2}$ of the Stake ; therefore, by *Prop. 1.* their Hazard is worth $\frac{5}{16}$, leaving to *C* and *D* $\frac{11}{16}$.

A and B playing at Whist against C and D; A and B have eight of ten, and C and D nine, and therefore can't reckon Honours; to find the proportion of their Hazards.

There

There is $\frac{5}{18}$ due to *A* and *B* upon their hazard of having three of four Honours; but since *C* and *D* want but one Game, and *A* and *B* two, there is due to *A* and *B* but $\frac{1}{4}$, or $\frac{4}{18}$ more upon that account, by *Prop.* 4. this in all makes $\frac{9}{18}$, leaving to *C*, and *D* $\frac{7}{18}$; so the hazard of *A* and *B* to that of *C* and *D*, is as 9 to 7.

In the former Calculations I have abstracted from the small difference of having the Deal, and being Seniors.

All the former Cases can be calculated by the *Theorems* laid down by Monsieur *Huygens*; but Cases more compos'd require other Principles: for the easy and ready Computation of which, I shall add one *Theorem* more, demonstrated after Monsieur *Huygens's* Method.

THEOREM.

If I have *p* Chances for *a*, *q* Chances for *b*, and *r* Chances for *c*, then my hazard is worth $\frac{ap + bq + cr}{p + q + r}$; that is, *a* multiplied into the number of its Chances added to *b*, multiplied into the number of its Chances, added to *c*, multiplied into the number of its Chances, and the Sum divided by the Sum of Chances of *a*, *b*, *c*.

To investigate as well as demonstrate this *Theorem*, suppose the value of my hazard be x , then x must be such, as having it, I am able to purchase as good a hazard again in a just and equal Game. Suppose the Law of it be this, That playing with so many Gamesters as, with myself, make up the number $p + q + r$, with as many of them as the number p represents, I make this bargain, that whoever of them wins shall give me a , and that I shall do so to each of them if I win; with the Gamesters represented by the number of q , I bargain to get b , if any of them win, and to give b to each of them, if I win myself; and with the rest of the Gamesters, whose number is $r - 1$, I bargain to give, or to get c after the same manner; Now all being in an equal probability to gain, I have p Chances to get a , q Chances to get b , and $r - 1$ Chances to get c , and one Chance, *viz.* when I win myself, to get $px + qx + rx - ap - bq - rc + c$, which, if it be suppos'd equal to c , then I have p Chances for a , q Chances for b , and r Chances for c (for I had just now $r - 1$ Chances for it) and therefore, in case $px + qx + rx - ap - bq - rc + c = c$, then is

$$x = \frac{ap + bq + cr}{p + q + r}.$$

By

HAZARDS of Game. 45

By the same way of reasoning you will find, If I have p Chances for a , q Chances for b , r Chances for c , and s Chances for d , that my hazard is $\frac{ap + bq + cr + ds}{p + q + r + s}$, &c.

IN NUMBERS.

If I had two Chances for 3 Shillings, four Chances for 5 Shillings, and one Chance for 9 Shillings, then, by this Rule, my hazard is worth 5 Shillings; for $\frac{2 \times 3 + 4 \times 5 + 1 \times 9}{7} = 5$; and it is easy to prove, that with five Shillings I can purchase a like hazard again; for suppose I play with six others, each of us staking 5 Shillings; with two of them I bargain, that if either of them win, he must give me 3 Shillings, and that I shall do so to them; and with the other four I bargain just so, to give or to get 5 Shillings: This is a just Game, and all being in an equal probability to win; by this means I have two Chances to get 3 Shillings, four Chances to get 5 Shillings, and one Chance to get 9 Shillings, *viz.* when I win my self; for then out of the Stake, which makes 35 Shillings, I must give the first two 6 Shillings, and the other four 20 Shillings, so there remains just 9 to myself.

It

It is easy, by the help of this *Theorem*, to calculate in the Game of Dice, commonly call'd *Hazard*, what Mains are best to set on, and who has the Advantage, the Caster or Setter. The *Scheme* of the Game, as I take it, is thus ;

Mains.	Throws next following for	
	The Caster.	The Setter.
V.	V.	II. III. XI. XII.
VI.	VI. XII.	XI. II. III.
VII.	VII. XI.	XII. II. III.
VIII.	VIII. XII.	XI. II. III.
IX.	IX.	II. III. XI. XII.

By an easy Calculation you will find, if the Caster has IV, and the Setter VII, there is due to the Caster $\frac{1}{3}$ of the Stake ; if he has

V against VII, $\frac{2}{3}$ of the Stake.

VI against VII, $\frac{5}{11}$ of the Stake.

IV against VI, $\frac{3}{8}$ of the Stake.

V against VI, $\frac{4}{9}$ of the Stake.

IV against V, $\frac{3}{7}$ of the Stake.

I need not tell the Reader, that IV is the same with X, V with IX, and VI with VIII.

Suppose

HAZARDS of Game. 47

Suppose then VII be the Main: To find the proportion of the hazard of the Caster to that of the Setter.

By the Law of the Game, the Caster, before he throws next, has four Chances for nothing, *viz.* these II, III, XII; eight Chances for the whole Stake, *viz.* those of VII, XI; six Chances for $\frac{1}{3}$, *viz.* those of IV, X; eight Chances for $\frac{2}{3}$, *viz.* those of V, IX; and ten Chances for $\frac{5}{11}$, *viz.* these of VI, VIII; so his hazard, by the preceding *Theorem*, is

$$\frac{4 \times 0 + 8 \times 1 + 6 \times \frac{1}{3} + 8 \times \frac{2}{3} + 10 \times \frac{5}{11}}{36}$$

Now to save the trouble of a tedious reduction, suppose the Stake which they play for be 36, that is, the Setter had laid down 18; in that case, every one of these Fractions are so many parts of an Unit, which being gather'd into one Sum, give $17\frac{4}{3}$ to the Caster, leaving $18\frac{1}{3}$ to the Setter; so the hazard of the Caster is to that of the Setter 244, 251.

Suppose VI, or VIII, be the Main, then the Share of the Caster is

II.

III. VI. IV. V.

XI. XII. X. IX. VIII. VII.

$$5 \times 0 + 6 \times 1 + 6 \times \frac{1}{2} + 8 \times \frac{2}{3} + 5 \times \frac{1}{3} + 6 \times \frac{6}{11} = 17\frac{3}{22},$$

leaving

leaving to the Setter $18\frac{1}{3}\frac{6}{9}\frac{7}{8}$; so the hazard of the Caster is to that of the Setter as 6961 to 7295.

Suppose V, or IX be the Main, then the Share of the Caster is

II.

III.

XI.

IV.

VI.

XII. V. X. IX. VIII. VII.

$$6 \times 0 + 4 \times 1 + 6 \times \frac{3}{7} + 4 \times \frac{1}{3} + 10 \times \frac{5}{9} + 6 \times \frac{3}{5} = 17\frac{1}{3}\frac{1}{3}\frac{2}{3},$$

leaving to the Setter $18\frac{8}{3}\frac{6}{15}$; so the hazard of the Caster is to that of the Setter as 1396 to 1439.

It is plain, that in every case the Caster has the Disadvantage, and that V, or IX, are better Mains to set on than VII, because, in this last Cast, the Setter has but 18 and $\frac{1}{3}\frac{4}{5}$, or $\frac{8}{3}\frac{4}{5}$; whereas, when V or IX is the Main, he has $18\frac{8}{3}\frac{6}{15}$; likewise VI, or VIII, are better Mains than V, or IX, because $\frac{1}{3}\frac{6}{9}\frac{7}{8}$ is a greater Fraction than $\frac{8}{3}\frac{6}{15}$.

All those Problems suppose Chances, which are in an equal probability to happen; if it should be suppos'd otherwise, there will arise variety of Cases of a quite different

different nature, which, perhaps, 'twere not unpleasant to consider: I shall add one *Problem* of that kind, leaving the Solution to those who think it merits their pains.

In Parallelipipedo cujus latera sunt ad invicem in ratione a, b, c: Invenire quotâ vice quivis suscipere potest, ut datum quodvis planum, v.g. a b jaciat.



HAZARDS of Game. 49

different nature, which, perhaps, were not
 supposed to consider: I shall add one I re-
 spect of that kind, leaving the solution to
 those who think it merits their pains.

In Travellers' tales, as in letters from ad-
 venturers, it is common to find a great deal
 of exaggeration, and the same is true of
 the history of the world, as in the case of
 the ancient historians, who were often
 misled by the reports of travellers.



A

II

A
DEMONSTRATION
OF THE
Gain of the BANKER

In any Circumstance of the
Game call'd *Pharaon*,

And how to determine the
Odds at the ACE of HEARTS,
or FAIR CHANCE ;

With the ARITHMETICAL SOLUTION of
some Questions relating to LOTTERIES:

- And a few Remarks upon HAZARD and
BACKGAMMON.

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Baccarat.



Of the Game of PHARAON.

IN order to demonstrate the Odds or Gain of the Banker in any circumstance of the Game of Pharaon, it will be necessary to solve the following Problem.

PROB. I.

Two Persons A, B, out of a heap of 9 Cards, four of which are red and five black, undertake to draw a red one blindfold, and he shall be reputed to win that draws the first: Now supposing A to have the first choice, B the second, A the third, and so on by turns till one of them wins; Quære, The proportion of their Hazards?

SOLUTION.

Let n be the number of all the Cards, r the number of red ones, b the number of black

black ones, and 1 the whole Stake, or the Sum play'd for.

1^o, Since *A* has *r* Chances for a red Card, and *b* Chances for a black one, it follows by the third Proposition that his Expectation is worth $\frac{r}{r+b}$ or $\frac{r}{n}$ of the Stake, equal to $\frac{r}{n} \times 1$; and accordingly let it be agreed between the two Gamesters, that instead of *A*'s attempting to draw a red Card he shall actually take out a black one, and as an equivalent shall have $\frac{r}{n}$ paid him out of the Stake, which being done, there will remain $1 - \frac{r}{n} = \frac{n-r}{n} = \frac{b}{n}$.

2^o, Since the remaining Cards are *n*—1, and *B* has *r* Chances for a red Card, it follows that his Probability of winning will be $\frac{r}{n-1}$, and consequently his Expectation upon the remaining Stake $\frac{b}{n}$ will be $\frac{r}{n-1} \times \frac{b}{n} = \frac{rb}{n \times n-1}$. But instead of *B*'s drawing, we will suppose the Sum $\frac{rb}{n \times n-1}$ paid him out of the Stake, and that a black Card being taken out of the heap as before, then the remaining Stake will be $\frac{b}{n} - \frac{rb}{n \times n-1}$ or $\frac{b \times n-1}{n \times n-1} - \frac{b \times n-b}{n \times n-1} = \frac{b \times b-1}{n \times n-1}$.

3^o, Now it comes to *A*'s turn again to chuse; and since he has still *r* Chances for a red Card, and the number of remaining Cards are $n - 2$, his Probability of winning this time will be $\frac{r}{n-2}$, and his Expectation

upon the remaining Stake $\frac{b \times b - 1}{n \times n - 1}$ will be

$\frac{b \times b - 1}{n \times n - 1} \times \frac{r}{n - 2}$, which we will likewise

suppose to be paid him out of the Stake,

and then there will remain $\frac{b \times b - 1}{n \times n - 1} -$

$\frac{b \times b - 1 \times r}{n \times n - 1 \times n - 2}$, or $\frac{b \times b - 1 \times n - 2}{n \times n - 1 \times n - 2} -$

$\frac{b \times b - 1 \times n - b}{n \times n - 1 \times n - 2} = \frac{b \times b - 1 \times b - 2}{n \times n - 1 \times n - 2}$, out of

which *B* may have $\frac{b \times b - 1 \times b - 2 \times r}{n \times n - 1 \times n - 2 \times n - 3}$;

and so we may proceed till the whole Stake is exhausted.

But I hope what has been said is sufficient to see the Law of Continuation, and form

this general Series; viz. $\frac{r}{n} + \frac{b}{n-1}P +$

$\frac{b-1}{n-2}Q + \frac{b-2}{n-3}R + \frac{b-3}{n-4}S, \&c.$ In which

'tis evident that *P*, *Q*, *R*, *S*, &c. denote the preceding Terms. Now to apply this

Series to practice, we must take as many Terms of it, as there are Units in $b + 1$;

for since *b* represents the number of black Cards, the number of Drawings cannot

exceed $b + 1$; therefore take for *A* the first, third,

third, fifth, &c. Terms; and for *B*, the second, fourth, sixth, &c. Terms, and the Sum of those Terms will be the respective Expectations of *A*, *B*; or because the Stake is fix'd, these Sums will be proportional to their respective Probabilities of winning. For instance, if we apply this to the present Case, the general Series will be $\frac{4}{9} + \frac{5}{8}P + \frac{4}{7}Q + \frac{3}{6}R + \frac{2}{5}S + \frac{1}{4}T$. And to bring this Series into whole Numbers, let us assume x a whole Number, which multiply'd by $\frac{4}{9}$ shall be a whole Number $= P$. Therefore since $\frac{4x}{9} = P$, $\frac{4x}{9} \times \frac{5}{8} = Q$, and $\frac{4x}{9} \times \frac{5}{8} \times \frac{4}{7} = R$, &c. each of which Terms are to be whole Numbers: Now 'tis evident that the Denominator of the Fraction represented by *P*, is an aliquot part of that represented by *Q*, and every Denominator an aliquot part of the next following, and so the last Denominator $9 \times 8 \times 7 \times 6 \times 5 \times 4$ is a Multiple to all the preceding Denominators; and consequently if each Numerator be multiplied by it, instead of x , and the Products divided by their respective Denominators, the Quotients will all be whole Numbers, this is universally true: But in this Case, if the last Fraction $\frac{4x \times 5 \times 4 \times 3 \times 2 \times 1}{9 \times 8 \times 7 \times 6 \times 5 \times 4}$ be reduced to its least Terms, viz. $\frac{x}{126}$, its Denominator 126 is the least common

common Multiple to all the rest of the Fractions when reduc'd to their least Terms.

Which being determin'd, the Terms of the Series are easily found by the following Operation to be $56 + 35 + 20 + 10 + 4 + 1$.

OPERATION.

$$\begin{array}{r} 126 \\ 4 \\ \hline 504 \\ 9) \hline \end{array}$$

$56 = P$

$$\begin{array}{r} 5 \\ \hline 280 \end{array}$$

8) —

$35 = Q$

$$\begin{array}{r} 4 \\ \hline 140 \end{array}$$

7) —

$20 = R$

$20 = R$

$$\begin{array}{r} 3 \\ \hline 60 \end{array}$$

6) —

$10 = S$

$$\begin{array}{r} 2 \\ \hline 20 \end{array}$$

5) —

$4 = T$

$$\begin{array}{r} 1 \\ \hline 4 \end{array}$$

4) —

$1 = U$

Wherefore assigning to A $56 + 20 + 4 = 80$

And to B ----- $35 + 10 + 1 = 46$

And their Probabilities of winning will be as 80 to 46, or as 40 to 23. And if there be never so many Gamesters $A, B, C, D,$ &c. the Probabilities of winning may as easily be assign'd by the general Series, as for these two.

REMARK.

The preceding Series may in any particular Case be easily shorten'd; for if $r=1$, then the Series will be

$$\frac{1}{n} \times 1 + 1 + 1 + 1 + 1 + 1, \text{ \&c.}$$

If $r=2$, then the Series will be $\frac{2}{n \times n - 1}$
 $\times n - 1 + n - 2 + n - 3 + n - 4, \text{ \&c.}$

If $r=3$, then the Series will be $\frac{3}{n \times n - 1 \times n - 2}$
 $\times n - 1 \times n - 2 + n - 2 \times n - 3 + n - 3 \times n - 4$
 $\text{\&c.}$

If $r=4$, then the Series will be $\frac{4}{n \times n - 1 \times n - 2 \times n - 3}$
 $\times n - 1 \times n - 2 \times n - 3 + n - 2 \times n - 3 \times n - 4$
 $\text{\&c.}$

Wherefore rejecting the common Multipliers, the several Terms of those Series taken in due order will be proportional to the several Expectations of any number of Gamesters. Thus in the Case of this Problem, where $n=9$, $r=4$, $b=5$, the Terms of the Series will be

For A.	For B.
$8 \times 7 \times 6 = 336$	$7 \times 6 \times 5 = 210$
$6 \times 5 \times 4 = 120$	$5 \times 4 \times 3 = 60$
$4 \times 3 \times 2 = 24$	$3 \times 2 \times 1 = 6$
480	276

Hence

Hence it follows, that the Probabilities of winning will be respectively as 480 to 276, or dividing both by 12, as 40 to 23, the same as before.

The Game of PHARAON.

Rules of the Play.

First, The Banker holds a Pack of 52 Cards.

Secondly, He draws the Cards one after the other, laying them alternately to his right and left-hand.

Thirdly, The Ponte may, at his choice, set one or more Stakes, upon one or more Cards taken out of his parcel of 13 Cards, from the Ace to the King inclusive, call'd a Book; either before the Banker has begun to draw the Cards, or after he has drawn any number of Couples, which are commonly call'd Pulls.

Fourthly, The Banker wins the Stake of the Ponte, when the Card of the Ponte comes out in an odd place on his right-hand; but loses as much to the Ponte when it comes out in an even place on his left-hand.

Fifthly, The Banker wins half the Ponte's Stake, when in the same pull the Card of the Ponte comes out twice.

Sixthly, When the Card of the Ponte, being once in the Stock, happens to be the last, the Ponte neither wins nor loses.

Seventhly, The Card of the Ponte being but twice in the Stock, and the two last Cards happening to be his Cards, he then loses his whole Stake.

P R O B. II.

To find at Pharaon the Gain of the Banker, when any number of Cards remain in the Stock; having the number of times that the Ponte's Card is contain'd in it, given.

W Herefore this Problem admits of four Cases, since the Ponte's Card may be contain'd either once, twice, thrice, or four times in the Stock.

Solution of the first CASE.

The Banker, according to the Law of the Game, has the following number of Chances for winning and losing; viz.

1	1	Chance for winning	1
2	1	Chance for losing	1
3	1	Chance for winning	1
3	1	Chance for losing	1
5	1	Chance for winning	1
*	1	Chance for losing	0

Hence

Hence the Banker has one Chance more for winning than for losing, and the number of all the Chances are equal to n , the number of Cards in the Stock; therefore $\frac{1}{n}$ is the Gain of the Banker upon the Stake. To illustrate this Case,

Let it be required to find the Gain of the Banker, when there are 20 Cards remaining in the Stock, and the Ponte's Card but once in it, $\frac{1}{n} = \frac{1}{20}$.

Answer, He gains the twentieth part of his Stake.

Solution of the second CASE.

By the Remark belonging to the preceding Problem, it appears, that the Chances which the Banker has to win or lose his Stake would be proportional to these Numbers $n - 1$, $n - 2$, $n - 3$, &c. were the Banker not allow'd half the Stake upon drawing of Doublets, or the Ponte's Card twice together; upon which account the number of Chances represented for the Banker by $n - 1$ for winning, must be divided into two parts $n - 2$, and 1, whereof the first is proportional to the probability which the Banker has for winning the whole Stake of the Ponte, and the second is proportional to the probability of winning the half of it: for since the Banker is not intitled to the whole

whole Stake upon the Ponte's Card coming out in an odd place, till he knows whether the next Card be the Ponte's, his Chance for winning the whole Stake can be no greater than the Ponte's; because there is the same reason, and easily prov'd, that the second Card, and not the first, should be the Ponte's, as the first Card, and not the second. Wherefore the Chances that the Banker and the Ponte have to win the whole Stake are equal, with respect to the order of drawing, and consequently should be express'd by the same number; whence arises this Scheme for determining the number of Chances which the Banker has for winning and losing the Stake, and the number of Chances for winning the half Stake, represented by y .

S C H E M E.

1	$\left\{ \begin{smallmatrix} n-2 \\ 1 \end{smallmatrix} \right\}$	Chances for winning	$\left\{ \begin{smallmatrix} 1 \\ y \end{smallmatrix} \right\}$
2	$n-2$	Chances for losing	1
3	$\left\{ \begin{smallmatrix} n-4 \\ 1 \end{smallmatrix} \right\}$	Chances for winning	$\left\{ \begin{smallmatrix} 1 \\ y \end{smallmatrix} \right\}$
4	$n-4$	Chances for losing	1
5	$\left\{ \begin{smallmatrix} n-6 \\ 1 \end{smallmatrix} \right\}$	Chances for winning	$\left\{ \begin{smallmatrix} 1 \\ y \end{smallmatrix} \right\}$
6	$n-6$	Chances for losing	1
*	1	Chance for winning	1

First,

First, $\frac{n-2}{2} =$ Number of Chances for gaining y from the nature of the Scheme, and $n \times \frac{n-1}{2} =$ Sum of all the Chances.

Therefore $\frac{n-2}{n \times n-1}$ is the probability of the Banker's gaining y , and $\frac{n-2}{n \times n-1} \times y =$ the Banker's Gain upon y .

Secondly, The Banker has but 1 Chance more to win the Stake than to lose; the rest for winning and losing being equal destroy each other. Therefore $\frac{1}{n \times \frac{n-1}{2}}$ is the pro-

bability of getting it, and $\frac{1}{n \times \frac{n-1}{2}} \times 1 =$
 $\frac{2}{n \times n-1}$ is his Gain upon the Stake.

Therefore the Banker's Gain upon the whole is $\frac{n-2}{n \times n-1} \times y + \frac{2}{n \times n-1} = \frac{n-2 \times y + 2}{n \times n-1}$
 $= \frac{\frac{1}{2}n + 1}{n \times n-1}$, if $y = \frac{1}{2}$.

To illustrate this Case by an Example in Figures, Let it be required to find the Gain of the Banker when there are 20 Cards remaining in the Stock, and the Ponte's Card twice in it.

$$\frac{\frac{1}{2}n + 1}{n \times n-1} = \frac{10 + 1}{20 \times 19} = \frac{11}{380} = \frac{1}{34} \text{ nearly.}$$

Answer, About the thirty-fourth part of his Stake.

That

That $n \times \frac{n-1}{2}$ is the Sum of all the Chances, may be thus prov'd. 'Tis evident by inspection of this Scheme, that they are equal to this Series $n-1, n-2, n-3, n-4, \&c.$ And $\frac{2}{n \times n-1} \times n-1 + n-2 + n-3, \&c.$ is a Series belonging to the preceding Problem, which expresses the Sum of the Probabilities of winning, which belong to the two Gamblers, when the number of all the Cards is n , and the number of red ones two. It therefore expresses likewise the Sum of the Probabilities of winning which belong to the Ponte and Banker in the present Case, there being two of the Ponte's Cards in the Stock; but the Sum of these Probabilities of winning are equal to Unity, because the Numerators of those Fractions which express their respective Probabilities of winning being added together, is equal to the common Denominator, and so equal to an Unit, consequently

$\frac{2}{n \times n-1} \times n-1 + n-2 + n-3, \&c.$ is equal to an Unit; wherefore putting $f = n-1, n-2, n-3, \&c.$ we shall have $\frac{2}{n \times n-1} \times f = 1$, and therefore $f = n \times \frac{n-1}{2}$.

Solution

Solution of the third CASE.

By the preceding Problem, the number of Chances the Banker has for winning or losing the Stake would be proportional to these Numbers $n - 1 \times n - 2$, $n - 2 \times n - 3$, &c. but that on account of the Doublets, the Banker's Chances for winning, as in the preceding Case, are divided into two parts: For instance, $n - 1 \times n - 2$ is divided into the parts $n - 2 \times n - 3$, and $2 \times n - 2$; the former of which is proportional to the Chances that both Banker and Ponte have to win the whole Stake the first pull, because there is the same probability that the Ponte's Card should be drawn the second time and miss'd the first, as drawn the first time and miss'd the second; and the latter part, *viz.* $2 \times n - 2$ is proportional to the Chances for both first and second Cards being the Ponte's the first pull: and as $2 \times n - 2$ is easiest found by subtracting $n - 2 \times n - 3$ from $n - 1 \times n - 2$; so by subtracting $n - 4 \times n - 5$, the number of Chances that the Banker and Ponte each have, for winning the whole Stake the next pull from $n - 1 \times n - 4$, there will remain $2 \times n - 4$, the number of Chances for the third and fourth Card being the Ponte's, and so for the rest: whence is easily deduc'd the Method of

K

forming

forming the following Scheme, which shews what Chances the Banker has for winning and losing.

SCHEME.

1	$\left\{ \begin{array}{l} n - 2 \times n - 3 \\ 2 \times n - 2 \end{array} \right\}$	Ch. for win.	$\left\{ \begin{array}{l} 1 \\ y \end{array} \right\}$
2	$n - 2 \times n - 3$	Ch. for losing	1
3	$\left\{ \begin{array}{l} n - 4 \times n - 5 \\ 2 \times n - 4 \end{array} \right\}$	Ch. for win.	$\left\{ \begin{array}{l} 1 \\ y \end{array} \right\}$
4	$n - 4 \times n - 5$	Ch. for losing	1
5	$\left\{ \begin{array}{l} n - 6 \times n - 7 \\ 1 \times n - 6 \end{array} \right\}$	Ch. for win.	$\left\{ \begin{array}{l} 1 \\ y \end{array} \right\}$
*	$n - 6 \times n - 7$	Ch. for losing	1

Hence it appears, that there are no more Chances for winning the Stake than for losing it; so all the Banker's Advantage in this case is upon the half Stake, which depends upon drawing of Doublets at one of the Pulls, and all the Chances he has for that may be found thus.

By the above Scheme, $2 \times n - 2$ is the first Term of the Series to win y , 4 is the common difference of the rest of the Terms, and 4 the last Term; and so the Sum of all the Terms by the known Laws of Arithmetical Progression is $\frac{n \times n - 2}{2}$, which divided

vided by the Sum of all the Chances =
 $\frac{n \times n - 1 \times n - 2}{3}$ by the preceding Problem,
 gives $\frac{3}{2 \times n - 1}$ for the probability of win-
 ning y ; therefore $\frac{3}{2 \times n - 1} \times y$ is the Ban-
 ker's Gain, equal $\frac{3}{4 \times n - 1}$, supposing $y = \frac{1}{2}$.

EXAMPLE.

Query, The Banker's Gain when the Stock
 consists of 20 Cards, and the Ponte's Card
 thrice in it?

$$\frac{3}{4 \times n - 1} = \frac{3}{4 \times 19} = \frac{3}{76} = \frac{1}{25} \text{ nearly.}$$

Answer, About the twenty-fifth part of
 his Stake.

Solution of the fourth CASE.

SCHEME.

1	$\left\{ \begin{array}{l} n-2 \times n-3 \times n-4 \\ 3 \times n-2 \times n-3 \end{array} \right\}$	Ch. for win.	$\left\{ \begin{array}{l} 1 \\ y \end{array} \right\}$
2	$n-2 \times n-3 \times n-4$	Ch. for losing	1
3	$\left\{ \begin{array}{l} n-4 \times n-5 \times n-6 \\ 3 \times n-4 \times n-5 \end{array} \right\}$	Ch. for win.	$\left\{ \begin{array}{l} 1 \\ y \end{array} \right\}$
4	$n-4 \times n-5 \times n-6$	Ch. for losing	1
*	$\left\{ \begin{array}{l} 2 \times 1 \times 0 \\ 3 \times 2 \times 1 \end{array} \right\}$	Chance for winning	$\left\{ \begin{array}{l} 1 \\ y \end{array} \right\}$
	$2 \times 1 \times 0$	Chance for losing	1

Here also the Chances for winning and losing the Stakes are equal, so that the Banker's Gain depends upon y , the half Stake.

And the Chances to win y are

$$\text{Equal to } \left\{ \begin{array}{l} 3 \times 2 \times 1 \\ 3 \times 4 \times 3 \\ 3 \times 6 \times 5 \\ \&c. \end{array} \right\} = 3 \times \left\{ \begin{array}{l} 2 \times 1 \\ 4 \times 3 \\ 6 \times 5 \\ \&c. \end{array} \right.$$

And to find the Sum of those Products, *viz.* $2 \times 1 + 4 \times 3 + 6 \times 5$, &c. continued to any number of Terms, whose Factors are in Arithmetical Progression, we must premise the following

LEMMA.

Subtract the second Product from the third, and the third Product from the fourth, and call the Remainders first Differences; then subtract those Differences from each other, and call the Remainder a second Difference. See the following Scheme.

$$\begin{array}{rcl} 2 \times 1 & = & 2 \\ \hline 4 \times 3 & = & 12 \\ 6 \times 5 & = & 30 \\ 8 \times 7 & = & 56 \end{array} \quad \begin{array}{r} \\ 18 \\ 26 \end{array} \quad \begin{array}{r} \\ \\ 8 \end{array}$$

Now if we call the first Product a , the second b , the first of the first Differences d' , the second Difference d'' ; and if the number

ber of Products which follow the first be called x , the Sum of all the Products will be equal to $a + x \times b + \frac{x}{1} \times \frac{x-1}{2} \times d' + \frac{x}{1} \times \frac{x-1}{2} \times \frac{x-2}{3} \times d''$.

Or

$$2 + x \times 12 + \frac{x}{1} \times \frac{x-1}{2} \times 18 + \frac{x}{1} \times \frac{x-1}{2} \times \frac{x-2}{3} \times 8.$$

Or

$2 + 5\frac{1}{3}x + 5xx + 1\frac{1}{3}xxx$, which multiplied by 3, the common Multiplier to all the Terms, the Product $6 + 17x + 15x^2 + 4x^3$ will be the Sum of all the Chances for winning of y , whose component Parts, by Sir *Iaac Newton's* Method of the Invention of Divisors, is readily found to be $2x + 2$, $2x + 4$, and $\frac{4x+3}{4}$; wherefore $\frac{2x+2 \times 2x+4 \times 4x+3}{4}$ is the number of

Chances for winning of y . Now $x =$ to the number of Terms to be added, bating 1, therefore make $x + 1 = p$, equal to the number of Terms, and then the Chances for winning of y are $\frac{2p \times 2p+2 \times 4p-1}{4}$. But according to the Scheme it appears, that the number of Terms to be added are equal to $\frac{n-2}{2}$; wherefore writing $\frac{n-2}{2}$ for p , and

and the Chances for winning are equal to $\frac{n \times n - 2 \times 2n - 5}{4}$, which divided by the Sum of all the Chances, *viz.* $\frac{n \times n - 1 \times n - 2 \times n - 3}{4}$

and the Quotient $\frac{2n-5}{n-1 \times n-3}$ is the probability of winning y , and so $\frac{2n-5 \times y}{n-1 \times n-3}$ is the Gain of the Banker, or $\frac{2n-5}{2 \times n - 1 \times n - 3}$ supposing $y = \frac{1}{2}$.

EXAMPLE.

Suppose the Stock to consist of 20 Cards, and the Ponte's Card four times in it; Query, The Banker's Gain?

$$\frac{2n-5}{2 \times n - 1 \times n - 3} = \frac{40-5}{2 \times 19 \times 17} = \frac{35}{646} = \frac{1}{18} \text{ nearly.}$$

Answer, $\frac{35}{646}$ of his Stake accurately, or the eighteenth part of it nearly.

A Table of Pharaon, whereby the several Advantages of the Banker, in whatever Circumstances he may happen to be, is seen sufficiently near by inspection, being calculated from the foregoing Theory.

N ^o of Cards in the Stock.	The N ^o of times the Ponte's Card is contain'd in the Stock.			
	1	2	3	4
52	**	**	**	50
50	**	94	65	48
48	48	90	62	46
46	46	86	60	44
44	44	82	57	42
42	42	78	54	40
40	40	74	52	38
38	38	70	49	36
36	36	66	46	34
34	34	62	44	32
32	32	58	41	30
30	30	54	38	28
28	28	50	36	26
26	26	46	33	24
24	24	42	30	22
22	22	38	28	20
20	20	34	25	18
18	18	30	22	16
16	16	26	20	14
14	14	22	17	12
12	12	18	14	10
10	10	14	12	8
8	8	11	9	6

But

But if an absolute degree of exactness be required, it will be easily obtain'd from the Rules and Examples given at the end of each Case. However, to make all things as plain as possible, I shall, to illustrate the Use of this Table, give an Example or two.

EXAMPLE 1.

Let it be required to find the Gain of the Banker when there are 30 Cards remaining in the Stock, and the Ponte's Card twice in it.

In the first Column seek for the Number answering to the number of Cards remaining in the Stock: over-against it, and under number 2, which is at the head of the Table, you will find 54, which shews that the Banker's Gain is the fifty-fourth part of his Stake.

EXAMPLE 2.

Let it be required to find the Gain of the Banker when there are but 10 Cards remaining in the Stock, and the Ponte's Card thrice in it.

Against 10, the number of Cards in the first Column, and under number 3, you will find 12, which denotes that the Banker's Gain in this Circumstance is the twelfth part of his Stake.

COROL-

COROLLARY 1. From the Construction of the Table it appears, that the fewer Cards there are in the Stock, the greater is the Gain of the Banker.

COROL. 2. The least Gain of the Banker under the same circumstance of Cards remaining in the Stock, is, when the Ponte's Card is but twice in it, the next greater when three times, still greater when but once, and the greatest of all when four times.





*Of the ACE of HEARTS, or FAIR
CHANCE.*

THIS Game is pretty much in vogue, as well as that of *Pharaon*; therefore it may not be improper here to touch a little upon the Advantage or Gain that accrues to the Banker or *Taliere* at this, as well as of that.

The manner of playing at it is as follows; There is a Table, on which is painted a select number of Cards, generally 31 or 25, or between those Numbers, limited at the fancy of the Person who banks the Table; the Player stakes upon either of such Cards more or less, at his pleasure. On the Table is fixed an Engine, called a Worm, into which is put an Ivory Ball, which runs round till it drops or falls into a Socket contiguous to one of those Cards; and if it happens to be the Card on which the Player has staked, he saves his Stake, and is intitled to 28 or 23 times as much more, according to the number of Cards painted on the Table, and the custom of the Place
where

where the Table is kept. From which Description of the Game, the following Problem naturally arises.

PROB. III.

The number of Cards upon such a Table, and the number of Stakes the Banker pays in case he loses, being given ; to find the Gain of the Banker upon any Sum deposited as a Stake.

RULE.

FROM the number of Chances or Cards the Table consists of, subtract the number of Stakes more by one than the Banker pays when he loses, and multiply the Remainder by the value of the Stake, and divide the Product by the number of all the Chances or Cards upon the Table, and the Quotient will be the Banker's Gain upon that Stake, and of the same Denomination with it ; consequently upon two Cards he will have twice that Advantage, upon three Cards thrice, &c. supposing the Stakes equal.

EXAM. I.

Suppose a Table consists of 31 Cards, and that the Banker pays 28 Stakes when he loses ; *Query*, His Advantage upon a Stake of 100 l ?

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31 The N^o of all the Chances.

29 The N^o of Stakes paid more
— by one.

2 Difference.

Multiply by 100 the Stake.

—
31)200 (6 l.

186

—
14

20

—
31)280 (9 s.

279

—
1

Answer, 6 l. 9 s. $\frac{1}{3}$.

But this Rule is capable of determining the Gain of the Banker when unequal, as well as equal Stakes are set upon two or more different Cards, it being only to be considered as so many distinct Operations.

EXAM. 2.

Suppose *A*, *B*, *C* play, and stake between them 100 l. viz. *A* 50, *B* 30, *C* 20; Query, The Advantage of the Banker?

1°,

1°, 31—29=2 Diff.

2°, 2 Difference.

$$\begin{array}{r} 50 \\ 31 \overline{) 100} (3 \text{ l.} \\ 93 \\ \hline 7 \\ 20 \\ \hline 31 \overline{) 140} (4 \text{ s.} \\ 124 \\ \hline 16 \end{array}$$

$$\begin{array}{r} 30 \\ 31 \overline{) 60} (1 \text{ l.} \\ 31 \\ \hline 29 \\ 20 \\ \hline 31 \overline{) 580} (18 \text{ s.} \\ 31 \\ \hline 270 \\ 248 \\ \hline 22 \end{array}$$

3°, 2 Diff.

$$\begin{array}{r} 20 \\ 31 \overline{) 40} (1 \text{ l.} \\ 31 \\ \hline 9 \\ 20 \\ \hline 31 \overline{) 180} (5 \text{ s.} \\ 155 \\ \hline 25 \end{array}$$

Hence the Banker's Gain upon $\left. \begin{array}{l} A \\ B \\ C \end{array} \right\}$ is $\left\{ \begin{array}{l} 3,04\frac{1}{3}\frac{6}{1} \\ 1,18\frac{2}{3}\frac{2}{1} \\ 1,05\frac{2}{3}\frac{5}{1} \end{array} \right.$

Total Gain $6,09\frac{1}{3}\frac{1}{1}$

The same as before; whence it follows, that let the Money be stak'd, how you will, that is, upon as many Cards in what shape soever you please, the Banker's Gain will be after the rate of 6 l. 9 $\frac{1}{3}$ $\frac{1}{1}$ per Cent. upon all the Money stak'd.

But

78 Of the ACE of HEARTS ;

But for the sake of a farther Illustration, let us suppose a Person to stake One Pound Sterling upon each Card, to the number of 31, viz. all the Cards upon the Table ; in such instance, 'tis plain, he's sure of winning, or to receive 28 *l.* for one of the Cards, and save the Stake of that Card which wins ; but then 'tis also evident upon the whole, that he must be a loser two Pounds, since he wins only 28, and loses 30. Wherefore by the Rule of Proportion it follows, that if in staking 31 Pounds he loses 2, in staking 100 Pounds he will lose 6 *l.* 9 $\frac{1}{3}$ *s.*, which consequently is the Banker's Gain. See the Operation.

$$\begin{array}{ccccccc} \textit{l.} & \textit{l.} & & \textit{l.} & \textit{l.} & \textit{s.} & \\ 31 : 2 :: 100 : 6. & 9\frac{1}{3} \end{array}$$

100

—

$$31 \overline{) 200} \text{ (6 } \textit{l.}$$

186

—

14

20

$$31 \overline{) 280} \text{ (9 } \textit{s.}$$

279

—

1

Wherefore

Wherefore the Advantage *per Cent.* is so plain, that 'tis needless to dwell any longer upon it: However, it may not be amiss to observe, that 'tis an equal Wager that any one Card will win once in 21 times, notwithstanding the number of all the Chances are 31; which Event is discover'd in this or any other Table, by this general

RULE.

From the number of all the Cards upon the Table, deduct one, and multiply the Remainder by seven tenths, and the Product is the Answer. So in this Case,

$$\begin{array}{r} 31 - 1 = 30 \\ \text{Multiply by } .7 \\ \hline 21,0 \\ \hline \end{array}$$

And the Product 21 is the number of Trials requisite for any one Card to win upon an equality of Chance.





Of LOTTERIES.

IN this place I shall consider the Solution of several Problems relating to Lotteries, which may be of use to prevent some mistakes that People, not vers'd in such Computations, frequently run into. For instance, in the present Lottery for the Year 1737, where the proportion of Blanks to the Prizes is as 9 to 1, 'tis natural enough to conclude, that 9 Tickets are requisite for the chance of a Prize; and yet from mathematical Principles 'tis evident, that 7 Tickets are more than sufficient for that purpose, that is, in 7 Tickets it is more likely to have a Prize than not: for this, and all other Cases of this nature, we shall give the Arithmetical Solution of the following Problem.

PROB. IV.

To find how many Tickets must be taken, to make it as probable that one or more Prizes may be taken as not.

RULE.

RULE.

Multiply the number of Blanks there are to one Prize by seven tenths, and the Product is the Answer.

EXAMPLE.

Query, The number of Tickets requisite in a Lottery, whereof the number of Blanks is to the number of Prizes as 9 to 1, to make it an equal Chance for one or more Prizes.

$$\begin{array}{r} 9 \\ .7 \\ \hline 6,3 \end{array}$$

Answer, The Product 6,3 shows there is more than an equality of Chance in 7 Tickets, but something less than an equality in 6.

EXAMPLE 2.

Query, The number of Tickets requisite in a Lottery, whereof the number of Blanks is to the number of Prizes as 5 to 1, to make it an equal Chance for one or more Prizes.

$$\begin{array}{r} 5 \\ .7 \\ \hline 3,5 \\ \hline M \end{array}$$

Answer,

Answer, The number of Tickets requisite to that effect is between 3 and 4.

P R O B. V.

To find how many Tickets must be taken, to make it as probable that two or more Prizes will be taken as not.

R U L E.

Multiply 1,678 always by the number of Blanks there are to a Prize, and the Product will be the Answer.

E X A M P L E.

How many Tickets must be had in a Lottery, to make it as probable that two or more Prizes will be taken as not, when there are 9 Blanks to a Prize?

$$\begin{array}{r}
 1,678 \\
 \times 9 \\
 \hline
 15,102
 \end{array}$$

Answer, More than 15 Tickets, or rather more than 16, as shall be prov'd farther on, tho' one might undertake upon an equality of Chance to have one at least in 7 Tickets.

The Numbers 0,7 and 1,678 made use of to solve this and the preceding Problem, is the result of determining the Limits of x in these Equations, viz. $a + b^x = 2b^x$,
and

and $\overline{a + b^x} = 2b^x + 2axb^{x-1}$, where a and b represent the number of Chances respectively for the happening and failing of an Event at any one trial.

'Tis not my design to give a Solution to both these Equations; but being determin'd to find the Value of x in one of them, I shall undertake the last, as apprehending it will be most acceptable, especially to those who are not very ready in solving exponential Equations.

$\overline{a + b^x} = 2b^x + 2axb^{x-1}$, becomes
 $\frac{\overline{a + b^x}}{b^x} = 2 + 2axb^{-1}$ by dividing both
 sides of the Equation by b^x . Make $a : b ::$
 $1 : q$, and the last Equation by substitution

becomes $1 + \frac{1}{q} = 2 + \frac{2x}{q}$, in which, if
 q be supposed equal to 1, x will be found
 equal to 3; but if q be supposed infinite,
 and $\frac{x}{q} = z$, x will be infinite also, and

we shall have $1 + \frac{1}{q} = 2 \times 1 + z$.

Again, the Hyperbolic Log. of $1 + \frac{1}{q}$
 multiplied by zq is equal to the Hyperbolic
 Log. of $2 +$ the Hyperbolic Log. of $1 + z$.
 But the Hyperbolic Log. of $1 + \frac{1}{q}$ multi-
 plied by $q = 1$, therefore $z =$ Hyperbolic
 M 2 Log.

Log. of 2 + the Hyperbolic Log. of $1 + z$.
 From which last Equation z may be found,
 for $z - \log. 1 + z = \log. 2 = ,693147$
 $= n$; but the Hyperbolic Log. of $1 + z$
 $= z - \frac{zz}{2} + \frac{z^3}{3} - \frac{z^4}{4}$, &c. therefore
 $\frac{z}{2} - \frac{z^3}{3} + \frac{z^4}{4} - \frac{z^5}{5}$, &c. $= n$, and by
 reverſing the Series $z = \sqrt{2n + \frac{2n}{3} +$
 $\sqrt{\frac{n^3}{162} - \frac{2nn}{135}}$, &c. $= 1,6777$, equal to
 $1,678$ nearly.

Hence the Value of x in all Caſes will
 be between $3q$ and $1,678q$; but x converges
 pretty ſoon to the laſt of thoſe Limits, and
 ſo the number $1,678$, when x is not too
 ſmall, gives the Answer ſufficiently exact;
 as in the following Example, where the
 Odds of the Event's happening is greater
 than in the former.

EXAMPLE 2.

Let it be required to find in how many
 Throws, one may undertake upon an equa-
 lity of Chance, to throw three Aces twice,
 with three Dice?

SOLUTION.

Out of the 216 Chances upon three Dice,
 there is but 1 Chance for three Aces, and
 215 againſt it; wherefore multiplying the
 above

above Number 1,678 by 215, and the Product 360,77 shows, that 360 Throws, or very near it, are requisite to produce the required effect.

But when x is small, as in the preceding Example, it needs a correction; for instead of 15,102 Tickets, it should be 16,443: which Correction is easily had by the Rule of double false Position. For being assur'd that x is found something too little, I therefore assume it equal to 16, and substitute it in

the Equation $1 + \frac{1}{q} = 2 + \frac{2x}{q}$ and find the left-hand side thereof less than the right by 0,1589; wherefore I increase the value of x four tenths more, *viz.* to 16,4, and substitute it in the Equation as before, and still find the left-hand side too little by 0,0155; then I multiply cross-ways, and proceed in the rest of the Operation according to the nature of the Rule, and find $x = 16,443$.

However, we are not destitute of a Method whereby the true value of x , and consequently that of n may be found directly, by the help of an infinite Series, *viz.* $\frac{s}{r}$

$+ \frac{s^2}{18r^3} + \frac{1+2r}{162r^3}s^3$, &c. For putting the

Hyperbolic Log. of $1 + \frac{1}{q} = m$, $mq - \frac{1}{2} = r$, the standing Quantity 1,791759

$= n$, and $2mq - n = s$. The first Term of the Series, *viz.* $\frac{s}{r}$ subtracted from 2, will give the value of z in this case, true to two places of Decimals, *viz.* 1,83, whence $1,83 \times 9 = 16,47$ is the value of x , or true number of Tickets very near; for three Terms of the Series make $x = 16,44300$. And if x be smaller still, but so as not to have the number of Blanks to a Prize less than 4,1473 (which seldom or never happens in Lotteries) more places of Decimals will turn out true; in short, the above Series will determine the Value of z all Cases, when q is between 4,1473, and any other number how great soever.

Though, as it has been observ'd before, when q is any thing large, $1,678q$ gives x sufficiently near; for two Terms of this Series, in the case of throwing three Aces twice with three Dice, make $z = 1,686$, and consequently $x = 1,686 \times 215 = 362,49$, which is not two throws more than by the former Computation.

Note, The Solution of this Equation, *viz.*

$$\frac{x}{a + b} = 2b^x + 2xab^{x-1} + x \times x - 1 \times a^2b^{x-2}$$
 will give the value of x , in the Case of a triple Event; and the Solution of this, *viz.*
$$\frac{x}{a + b} = 2b^x + 2xab^{x-1} + x \times x - 1 \times a^2b^{x-2} + \frac{x}{1} \times \frac{x-1}{1} \times \frac{x-2}{3} a^3b^{x-3}$$
 will

will give the value of x in the Case of a quadruple Event. Here follows a Table of the Limits of x from one to six Events inclusive.

The Value of x will always be,

For a	{	single	}	Event	between	{	19	}	and	{	0,6939	}
		double					39				1,6789	
		triple					59				2,6759	
		quadruple					79				3,6729	
		quintuple					99				4,6709	
		sextuple					119				5,6689	

PROB. VI.

The Number of Tickets a Person has in a Lottery being given, to find the Odds against him whether they all prove Prizes.

RULE.

To the number of Blanks to a Prize add 1, and make the Sum the Denominator of a Fraction whose Numerator is Unity; then multiply this Fraction continually into itself as often as the Person has Tickets in the Lottery, bating one, and from the last Fraction thus produc'd, if Unity be taken from its Denominator, the Remainder will shew how many to 1 it is, that they all prove Prizes.

EXAMPLE.

EXAMPLE.

Suppose I have three Tickets in the Lottery of this Year 1737, where there are 9 Blanks to a Prize; how many is it to one but that they are all Prizes?

$9 + 1 = 10$ the Denominator.

$$\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1000}.$$

Answer, 999 to 1.

N. B. This Rule is only applicable to Lotteries, or in Schemes where there are a great number of Blanks and Prizes.

PROB. VII.

Having the Number of Tickets, and the Number and Amount of all the Prizes undrawn at any time given, to find the Value of a Horse for any number of Days.

RULE.

Multiply the number of Prizes by the Price of an undrawn Ticket, and subtract the Product from the Amount of all the Prizes, and multiply the Remainder by the number of Days the Horse is hired for, and reserve the Product for a Dividend; then multiply the number of undrawn Tickets by the number of Days required to draw them in, and with the Product divide the aforesaid Dividend, and the Quotient will be the Value of the Horse.

EXAMPLE.

Of LOTTERIES.

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EXAMPLE.

Let it be required to find the Value of a Horse for the first Day's drawing in the present Lottery, where there are 70000 Tickets at 10/ each, number of the Prizes 7000, amounting to 226000 l. exclusive of the two Prizes for the first and last Numbers drawn, viz. 500 and 1000 l. and let us suppose that the whole time of Drawing will be 40 Days.

The N ^o of Prizes	_____	7000
Price of an undrawn Ticket	_____	10
		<u>70000</u>

The N ^o of undrawn Tickets	_____	70000
The whole time of drawing	_____	40 d.
		<u>2800000</u>

The Amount of all the Prizes; deducting 14 per Cent. is 194360 l.

194360	
<u>70000</u>	
2800000	124360,000 (,044
<u>11200000</u>	20
	<u>,880</u>
12360000	12
<u>11200000</u>	<u>d.10,560</u>
	4
<u>11600000</u>	<u>f.2,240</u>

Answer, 10 d. $\frac{1}{4}$.

N

Those

Those that do not chuse to divide decimally, may, if they please, multiply the aforefaid Dividend by 960 before they divide, and the Quotient will be the Answer in Farthings; *e. g.*

$$\begin{array}{r}
 124360 \\
 \underline{960} \\
 7461600 \\
 1119240 \\
 \hline
 28)000000|1193|85600 \\
 \underline{42} \qquad \qquad \underline{112} \\
 4)\underline{\hspace{1cm}} \qquad \qquad 73 \\
 \quad 10\frac{1}{2} \qquad \qquad \underline{56} \\
 \qquad \qquad \qquad \underline{\hspace{1cm}} \qquad \qquad 17
 \end{array}$$

Which $10d. \frac{1}{2}$ would be the real Value of the Horfe; but as it's on the first Day's Drawing, there is a probability of its being the first drawn, in consequence of which the Owner is intitled to a Prize of 500 *l.* this Expectation is worth about three Half-Pence more, *viz.* the 70,000 part of the Value of 500 *l.* when the 14 *per Cent.* is taken off, which being added to the Value before found, makes 1 Shilling, the mathematical Value of a Horfe for the first Day's drawing.

EXAMPLE 2.

Admit that 12 Days before the end of the Drawing, there are left in the Wheel

Of LOTTERIES. 91

of Fortune 20544 Tickets, of which 2100 are Prizes, amounting in all to 71400 *l*.
Query, The Value of a Horfe for two Days, the Price of an undrawn Ticket at that time being worth 11 Pounds.

The Amount of all the Prizes, deducting 14 *per Cent*, is 61404 Pounds.

The Number of Prizes	_____	2100
The Price of an undrawn Ticket		<u>11</u>

2100

2100

23100

The Number of undrawn Tickets	20544
Number of Days to draw them in	<u>12</u>

246528

61404

23100

38304

2

246528)76608,000 (,310

739584

20

s. 6,200

264960

12

246528

d. 2,400

184320

4

1,600

Answer, 6 s. 2 d. $\frac{1}{4}$.

N 2

Note,

Note, In these Calculations 'tis supposed, (as is customary) that if the Horse proves a Prize during your Jockeyship, that 10 l. or an undrawn Ticket be restor'd to the Person who let it.

It is also very plain that this Rule will serve to value the Chance of a Ticket during the whole Lottery, assume the number of Days it will take in drawing what you please; wherefore to render the Operation easy, suppose 1 Day. And altho' I have made a deduction of 14 *per Cent.* upon the Amount of all the Prizes in the two preceding Examples, and at the same time allow the Calculation to be agreeable to the Rules of Art and Science, as being founded on the strictest Demonstration; yet in the present Case, I say, the Deduction is not quite reasonable between Buyer and Seller, and consequently none should be made in finding the value of a Chance for the whole time: For how can it be expected that any one will give up his right in a Ticket if it proves a Prize, and stand to the loss of a Guinea extraordinary if it proves a Blank, *viz.* the Discount upon 7 l. 10 s. at the rate of 14 *per Cent.* without a valuable consideration, which is that of taking Chances, valued according to the full Amount of the Prizes, and the Price the Tickets bore when they were first purchas'd, *viz.* 10 l. each.

This

This Method of proceeding will put both Parties upon an equal footing, than which, I think nothing can be more fair and equitable.

Hence I make the Chance of a Ticket for the whole time of drawing to be worth $2\text{ l. } 4\text{ s. } 6\text{ d. } \frac{6}{7}$, which, with $2\text{ d. } \frac{4}{7}$ for the Expectation of its being either the first or last drawn, makes $2\text{ l. } 4\text{ s. } 9\text{ d. } \frac{8}{7}$; but if the Chance happens to prove a Prize, 10 l. more, or the Price of an undrawn Ticket must be advanc'd.—However, the Market-Price determines what must be given after all; wherefore if a Chance should sell for more than what this Calculation makes, it is not to be wonder'd at, since some consideration ought to be made for the risque that the Dealers in Tickets run in having them sold under par, and for some contingent Expences they are unavoidably at, in furnishing those with Chances and Tickets who are willing to be in Fortune's Way.

But as in all Lotteries Success is precarious, we being kept in suspence till the Event makes known either our good or bad Fortune, so from such state of Uncertainty it follows, that before the Drawing is finish'd a Ticket may be sold for more or less than at present; I shall therefore, before I conclude this Subject, shew how its real worth may be known in any circumstance

cumstance of the Lottery, by which means the value of a Chance may be very accurately determin'd at the same time, *e. g.*

Multiply the Number of Blanks remaining in the Lottery at any time of its drawing, by the Price of a Blank, which is always fix'd, and to the Product add the Amount of all the Prizes remaining, the last drawn included; this Sum divided by the Number of all the Tickets, *viz.* Blanks and Prizes, will give the value of an undrawn Ticket, which being known, the Value of a Chance for the time, during the remaining part of the Lottery, easily flows from the afore-mentioned Rule,

I shall conclude this small Tract by making some Remarks relating to Hazard and Backgammon; the truth of all which is easily deduc'd from the preceding Propositions.



Of HAZARD.

1. **I**F 8 and 6 are Main and Chance, one may lay 155 to 169, or 11 to 12, that either one or the other is thrown off in two Throws.

2. And if 5 and 7, or 9 and 7 are Main and Chance, the probability of their being thrown off in two Throws is also as 155 to 169, or as 11 to 12.

3. If 5 and 8, or 9 and 8, or 5 and 6, or 9 and 6 are Main and Chance, the probability of throwing one of them off in two Throws is as 7 to 9 exactly.

4. And if 7 and 4, or 7 and 10 are Main and Chance, the probability of their being thrown off in two Throws is also as 7 to 9.

5. If 7 and 8, or 7 and 6 are Main and Chance, one may lay 671 to 625, or 15 to 14 that one of them is thrown off in two Throws, so he that lays an even Wager he will throw one of them off in two Throws has the best of the Lay.

6. But if 5 and 4, or 5 and 10, or 9 and 4, or 9 and 10 are Main and Chance; he that undertakes to throw either Main or Chance in three Throws has the worst of the Lay; for it is as 22267 to 24389, or in smaller Terms, as 21 to 23 exceeding near; the Ratio of 21 to 23 differing from that of 22267 to 24389 only but by the ten thousandth part of an Unit.

Note also, that 11 and 12 express the Ratio of 155 to 169 the nearest possible in such small Terms, as does 15 to 14 that of 671 to 625, and are easily discover'd by the Rule exhibited in my Appendix to Dr. Keil's Euclid.

7. Suppose IV to be a Main, and the Law of the Hazard to be this; That if the Caster throws either II, III, IV, XI, or XII the first Throw, he shall lose the whole Stake, and if he throws V, VI, VII, VIII, IX, or X, either of which, as it may happen, shall be deem'd a Chance against IV, so which ever comes up first wins; *Query*, The Hazard of the Caster to that of the Setter?

Answer, The Hazard of the Caster is to that of the Setter as 457 to 551, or as 5 to 6 very near; wherefore the Gain of the Setter, each Stake being a Guinea, will be $\frac{47}{304}$ equal to 1s. 11d. $\frac{1}{4}$ exactly.

8. And at Hazard, if the Main be 7, and each stake a Guinea, the Gain of the Setter is about $3d. \frac{1}{2}$.

9. If the Main be 6 or 8, the Gain of the Setter is about Six-pence in a Guinea.

10. But if the Main be 5 or 9, the Gain of the Setter is about $3d. \frac{3}{4}$ in a Guinea; whence it follows, that 5, 7 and 9 are much upon a par to set on, and that 6 and 8 are something more advantageous.

11. However, if a Person is determin'd to set upon the first Main that is thrown, his Advantage, supposing each Stake to be a Guinea, is the $\frac{37}{2016}$ of a Guinea, which when reduc'd will be found equal to $4d. \frac{1}{2}$, and half a Farthing exactly.

12. Hence the probability of a Main, to the probability of no Main; or, to speak in the gaming Phrase, a Main or no Main, is as $2016 - 37$ to $2016 + 37$; that is, as 1979 to 2053 accurately, or as 27 to 28 very near; for if one stakes 27 Guineas, the other ought not to stake quite $2d. \frac{1}{2}$ more than 28 Guineas, which is a small difference from the truth in such large Stakes as 27 and 28 Guineas.

13. If, with two Dice, one should undertake to throw first the two Aces, next the two Duces, next the two Threes, next the two Fours, next the two Fives, and lastly the two Sixes, the Odds against him would be

two thousand, one hundred, and seventy-six Millions, seven hundred, eighty-two thousand, three hundred, and thirty-five to one; and tho' this might possibly happen the first six throws, yet the Odds are so immensely great against it, that it would probably require whole Ages to perform it in: yet notwithstanding all this difficulty in throwing first the two Aces, next the two Duces, &c. they may with an equality of Chance be undertaken to be thrown in less than a quarter of an Hour, in the following manner, *viz.* to throw away till the two Aces come up, then till the two Duces, then till the two Threes, and so on till the two Sixes are thrown; but to throw them successively is what, never yet, is rational to suppose, has been done by any one.

14. If any one should undertake to throw a Six or an Ace with two Dice in one throw, he ought to lay 5 to 4; whereas 'tis usual to lay an even Wager only; in which circumstance the Caster has so much the better of the lay, as in the long run to impoverish the best Estates, not to say ruin them. Tho' at first sight it must appear to an Eye not vers'd in these Speculations, a little odd, that the Setter should not have the best of it, since there are but two Sixes, and two Aces for the Caster, and two Fives, two Fours, two Threes, and two Duces,
for

for the Setter. And were the Points of both Dice all made upon a regular Solid or Body of 12 equal Faces, such as the Dodecaedron, the Caster would undoubtedly have the advantage; for then he would have two to one of the lay, in as much as he would have 8 Chances for winning, and but 4 for losing: But as there are two Dice, it must be considered as the happening of two Events, independent of each other, which makes the Odds, as I said before, just 5 to 4.

COROLLARY.

Hence it follows, that at Backgammon if two Points are open, 'tis 5 to 4 but that a Person enters the first throw; and as this Thought naturally leads me to give a Solution of the rest of the Hazards, it may perhaps be acceptable if I shew the Odds of entering when other Points of the Table are open, and therefore shall give the following Scheme for that purpose.

SCHEME.

Points open	A Person may lay	
1	11	25
2	5	4
3	3	1
4	8	1
5	35	1

} to { that one enters.

And

100 *Of* BACKGAMMON.

And I don't doubt, but the Knowledge of these Odds may enable one to play the Game in other respects with great advantage; tho' for my part I own with regard to practice, that I have but very little skill in this, or any other Game whatsoever.

F I N I S.



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